TIME-DEPENDENT DEFORMATION CHARACTERISTICS OF COMPACTED COHESIVE SOILS

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BY

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JOINT HIGHWAY RESEARCH PROJECT

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INDIANA STATE HIGHWAY COMMISSION

Progress Report

TIME-DEPENDENT DEFORMATION CHARACTERISTICS OF

COMPACTED COHESIVE SOILS

TO: J. F. McLaughlin, Director
Joint Highway Research Project

May 27, 1970

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FROM: H. L. Michael, Associate Director
Joint Highway Research Project

Project No. C-36-5F

The attached Progress Report on the HPR research project "Long-Term Deformation of Compacted and Cohesive Soil Embankments" has been authored by Messrs. S. V. Ramaswamy and W. H. Perloff. The report is titled "Time-Dependent Deformation Characteristics of Compacted Cohesive Soils".

Progress on one phase of the research, characterization of the time-dependent mechanical behavior of compacted cohesive soils, is described in this report. The results will be very useful in the development of a model which will predict the long-term deformation of compacted clay embankments.

The report is presented to the Board for acceptance as partial fulfillment of the objectives of the research and will also be forwarded to the ISHC and BPR for review, comment and acceptance.

Respectfully submitted,

Harold L. Michael

Harold L. Michael
Associate Director

HLM/lm

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ABSTRACT

Experimental investigations of time-dependent shear strain characteristics of compacted cohesive soils under a step deviator stress load were conducted. It was found that by expressing creep parameters as a function of the ratio of the deviator stress to the deviator stress at failure the effects of confining pressure and moisture content could be accounted for.

It is shown that the response under second cycle of loading is more important for field structures and that the material constants obtained for small samples in the laboratory are valid for large structures also.

When the principal stress directions were changed, the change in stress-strain-time behavior of the compacted soils indicated that they behaved as anisotropic materials.

It was found that nonlinear viscoelastic constitutive equations may be used to predict the creep and recovery behavior of the compacted clays tested.

SECTION 1 - INTRODUCTION

1.1 - Introduction

This report describes progress on one phase of JHRP Project C-36-5F, "Long-Term Deformation of Compacted Cohesive Soil Embankments". The objective of the project is to develop a model for predicting long-term deformation of compacted clay embankments and to demonstrate that the model is a valid predictor under field conditions. The research described herein is that portion of the study relating to characterization of the time-dependent mechanical behavior of compacted cohesive soils.

1.2 - Importance of Material Characterization

Any descriptive model which purports to predict the deformation of a material body subjected to a set of internal and boundary loadings must include some description of the material of which the body is composed. Such a description, the material characterization or constitutive law, is necessarily a simplification of reality. But because the outcome of an analysis depends so markedly upon this description it is important to select a constitutive law which contains those factors germane to the problem at hand. Thus, the highly simplified assumptions of linear elastic theory may be useful for computing the distribution of vertical stresses in a cohesive soil mass. When considering deformations however, the time-dependent nature of clay soils must be incorporated into the material description.

This report describes results of laboratory studies to establish a useful description of the time-dependent mechanical behavior of compacted cohesive soils, which is incorporated in a suitable analysis of the embankment problem.

SECTION 2 - REVIEW OF LITERATURE

The ideas formulated by various authors on topics which are related to the time-dependent deformation of partly saturated soils are presented. The review is by no means exhaustive and an attempt is made only to present a general view of the current thinking.

2.1 Shear Deformation Studies of Cohesive Soils

Rheological models which can be visualized as linear springs in combination with linear or nonlinear dashpots have been extensively used to characterize the time-dependent shear deformation behavior of clays. Murayama and Shibata (1961) analyzed results of a number of triaxial creep tests taking into account the effect of temperature. Christensen and Wu (1964) have made use of the rate process theory of Eyring to determine the parameters of their rheological model and this gives a fairly accurate description of the creep behavior of clays under some ranges of loading.

Various experimental studies have been conducted to develop an understanding of the basic mechanisms contributing to the shearing resistance of soils and the factors that control the time-dependent responses to stress and strain. Mitchell (1964) used the rate process theory to relate the shearing resistance of soils in triaxial compression tests to frictional and cohesive parameters, effective stress, soil structure, rate of strain and temperature. He showed that his derived equation yielded reasonable results if reasonable values were assumed for the unknown parameters. Singh and Mitchell (1968) proposed a simple phenomenological relationship for the stress-strain-time behavior of soils, valid for a range of creep stresses varying from 30 per cent to 90 per cent or more of the initial strength, as follows:

$$\dot{\varepsilon} = A\, e^{\bar{\alpha}\bar{D}}\, (t_1/t)^m \qquad (2.1.1)$$

where

- $\dot{\varepsilon}$ = strain rate at any time t;
- A = projected value of strain rate at zero deviator stress on the logarithm strain rate versus deviator stress plot for unit time;
- α = value of slope of the mid-range linear portion of logarithm strain rate versus deviator stress plot;

$\bar{\alpha}$ = $\alpha\, D_{max}$;

D_{max} = strength of soil measured at the beginning of creep;

$\bar{D}$ = stress level = D/D_{max};

D = deviator stress $(\sigma_1 - \sigma_3)$;

t_1 = unit time;

t = any time, t;

According to them the same basic form of relationship is applicable, irrespective of whether the clays are undisturbed or remolded, wet or dry, normally consolidated or overconsolidated, or tested drained or undrained. But most of the data presented pertained to undrained tests and secondary compression part of the drained tests. Walker (1969) has questioned the validity of the proposed three parameter equation for the entire spectrum of macroscopic response of clay structure to an applied stress system. He felt that the parameters A, α, and m are likely to be functions of the testing conditions. He concluded that any generalized relationship should include all the variables involved in the deformation process (i.e.) volumetric strain, shear strain and pore water pressure and their variation with applied stress and time.

Mitchell and Campanella (1963a) and Mitchell, et al., (1969) used the rate process theory to develop a working hypothesis, which relates creep rate to soil properties and applied stress. However the practical utility of the expressions developed by them remains doubtful, since they have not indicated any reliable method for accurately evaluating the various parameters involved in their equations.

Lara-Tomas (1962) subjected hollow cylindrical specimens of compacted soil to constant shear stresses and studied their creep behavior. He concluded that suitable rheological models can be evolved to fit the data.

Perloff (1966) conducted extensive triaxial creep tests on compacted cohesive soils and obtained a relation between the shear strain and time as follows:

$$(\varepsilon_1 - \varepsilon_3) = at^b \qquad (2.1.2)$$

where

ε_1 = axial strain;

ε_3 = lateral strain;

a,b = constants;

He also concluded that "b" can be taken as essentially constant, irrespective of stress-level, confining pressure and moisture content and that by expressing "a" as a function of the ratio between the stress level and the failure stress, the effects of moisture content and confining pressure are removed.

Pagen and Jagannath (1967,1968) concluded that compacted cohesive soils can be treated as linear viscoelastic materials within a given range of stress and strain depending on the environmental condition and compaction methods. But they have used absolute numbers for their ranges of linear viscoelasticity and this severely restricts the applicability of their findings. They expressed the strain-time response by the relation,

$$\varepsilon = ct^m \tag{2.1.3}$$

where

ε = axial strain;

t = time;

c,m = constants.

Kondner (1963) suggested that the results of creep tests be reported in the form of creep compliances. According to Mitchell and Campanella (1963b), evaluation of creep compliance functions in case of non-linear viscoelastic materials may not provide much insight into creep mechanism. But nonlinear creep compliance functions have been extensively used in the analysis of polymer materials. Using thermodynamic principles, various expressions have been developed to characterize the non-linear viscoelastic materials and it is possible to take into account different kinds of nonlinearities in the formulation of these equations as will be discussed below.

Sankaran (1966) proposed a generalized stress-strain-time relationship for compacted soils relating octahedral shear stress and octahedral shear strain based on his test results. He had conducted one dimensional consolidation tests on saturated soils, and triaxial creep tests on partly saturated soils.

For principal stresses applied in the coordinate directions, the octahedral shear stress is,

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]^{\frac{1}{2}} \tag{2.1.4}$$

since $\sigma_2 = \sigma_3$ in both the triaxial and one-dimensional consolidation tests, we get

$$\tau_{oct} = \frac{\sqrt{2}}{3}(\sigma_1 - \sigma_3) \tag{2.1.5}$$

Also

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) \tag{2.1.6}$$

Similarly the octahedral shear strain and maximum shear strain are, respectively,

$$\gamma_{oct} = \frac{2\sqrt{2}}{3}(\varepsilon_1 - \varepsilon_3) \tag{2.1.7}$$

and

$$\gamma_{max} = (\varepsilon_1 - \varepsilon_3) \tag{2.1.8}$$

So it is obvious that a relationship postulated between octahedral shear stress and shear strain is equally valid for the maximum shear stress and strain. In one dimensional consolidation both volumetric strain and shear strain can be expressed in terms of axial strain as follows:

$$\text{volumetric strain} = \frac{\Delta v}{v} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_1 \tag{2.1.9}$$

since $\varepsilon_2 = \varepsilon_3 = 0$

$$\gamma_{max} = \varepsilon_1 - \varepsilon_3 = \varepsilon_1 \tag{2.1.10}$$

Hence it appears that additional experimental evidence is needed to make critical checks on the generality of the proposed relationships.

A similar argument applies to other investigations in which applicability of uniaxial test results to the description of general stress-strain-time response is assumed. Even for idealized linear materials such an assumption is generally unwarranted.

2.2 - Volume Change Characteristics of Partly Saturated Soils.

There have been numerous experimental and theoretical investigations of volume change characteristics of saturated soils since Terzaghi published his theory of one dimensional consolidation. However, consolidation of partly saturated soils is a relatively neglected subject.

Biot (1941) extended Terzaghi's theory of one dimensional consolidation to a general theory of three dimensional consolidation of a poro-elastic solid.

But no attempt was made to take into account the effects of included air on the process as a physical problem. Also the secondary compression effects and/or viscoelastic characteristics of the soil skeleton were neglected in his analysis.

Alpan (1961) introduced a modified time factor ρT in the Terzaghi equation to account for the consolidation of partly saturated soils. The factor ρ is a function of degree of saturation, change in volume of pore air and the change in volume of the soil sample. However he did not take into account the change in properties of the fluid and the skeleton during the process of consolidation.

Yoshimi and Osterberg (1963) performed extensive experiments on the consolidation of partly saturated soils. They concluded that there was virtually no outflow of pore water during compression, even when the degree of saturation was about ninety per cent. Since the pore water movement was negligible, the time rate of compression was governed by the rheological characteristics of the soil skeleton. Hence the time rate of compressive strain was independent of the drainage conditions or thickness of the specimen, but dependent on the stress increment ratio.

Teerawong (1963) investigated consolidation of soils compacted dry of optimum. He assumed that the air voids in the soil specimen were completely interconnected and only air escaped from the soils during consolidation. He obtained an expression for time-dependent compression taking into account the compressibility of air, solubility of air in water, and assuming viscous resistance only for the soil skeleton. Danielson (1963) extended this work to include permanent shearing resistance of the skeleton as well as viscous response.

Barden (1965) developed a series of nonlinear partial differential equations containing unknown functions for the one-dimensional consolidation of partly saturated soils, the range of degree of saturation varying from zero to one hundred per cent. Based on the degree of saturation, he classified any particular consolidation process into one of five idealized processes from extremely dry clays (degree of saturation less than fifty per cent) to very wet clays (degree of saturation greater than ninety per cent). To solve the equations, he assumed linear relationships between pore water pressure and permeability, between pore water pressure and density of pore fluid

and between pore water pressure and porosity. His results suggest that for clays compacted wet of optimum, the variation in permeability during consolidation has a more important effect on the process than the compressibility of the pore fluid.

The importance of degree of saturation to air permeability was shown by Langfelder, Chen and Justice (1968) who found that the air permeability of cohesive soils was affected by water content, dry density, and the method of compaction. The changes in air permeability as water content was increased were small on the dry side of optimum water content, until the water content reached the optimum moisture content, when it became nearly zero. They concluded that as the water content increased, the connecting necks between larger air pores tended to close and the continuous passage for air to flow through ceases to exist. The phenomenon of occlusion takes place and the air permeability becomes negligible.

Toriyama and Sawada (1968) studied the one dimensional consolidation characteristics of partly saturated soils compacted wet of optimum. They derived an equation which takes into account the compressibility of the pore fluid, changes in permeability with degree of saturation and effective stress, as well as changes in compressibility of the skeleton with effective stress. They solved the derived equation numerically, and found that the pore water pressure set up by loading depended on the compressibility of the soil skeleton and the degree of saturation.

The existence of air in addition to water in the pores of partly saturated soils makes the study of its behavior extremely complicated. Furthermore relatively few experimental investigations concerning volume change characteristics have been undertaken. The analytical efforts have been concerned with describing soil response under one-dimensional conditions. Without further information, however, the results of one-dimensional analyses and experiments cannot be extrapolated to the more general problem of interest herein.

2.3 - Pore Pressure in Compacted Soils.

Hilf (1956) conducted an extensive study of the pore fluid pressures in partly saturated soils and was the first to measure directly the negative pore water pressure in compacted soils. He developed the axis translation technique to permit measurement of capillary pressures less than one atmosphere below zero, and found a large negative pore water pressure in soils

compacted near the optimum water content. The axis translation technique, described in detail by Hilf (1956) has been extensively used since then.

Lambe (1961) investigated the variation of the magnitude of negative pore water pressure in compacted cohesive soils with degree of saturation. He concluded that the degree of saturation was an important factor in the magnitude of negative pore water pressure and that there was a marked decrease in the magnitude of the negative pressure as the degree of saturation was increased to one-hundred per cent.

Olsen and Langfelder (1965) have reported negative pore pressures as low as -100 psi for plastic clays compacted two per cent dry of optimum water content. Samples of Grundite clay compacted at 127 psi had pore water pressures lower than -250 psi when the moisture contents were only five per cent less than the optimum moisture content. They also pointed out that consideration of the osmotic pressures near clay particle surfaces lead to the conclusion that the actual pore water pressures in unsaturated soil varied from approximately -1 atmosphere to high positive values.

Schuurman (1966) assumed that pore air formed bubbles in pore water and obtained a relation between the pore air pressure u_a, and the pore water pressure u_w. He showed that as the air bubble size decreased, there is an increase in the value of $(u_a - u_w)$.

Matyas and Radhakrishna (1970) subjected cylindrical samples to all round pressure and to zero radial strain (K_o) loading. They concluded that the principle of effective stress was not adequate to explain the volumetric behavior of partly saturated soils subjected to different stress paths.

Except for the influence of the degree of saturation on the magnitude of initial negative pore water pressure, the influence of other factors like pore size distribution is unknown. Furthermore, neither the magnitude of developed pore water pressure in response to loading, nor the relationship between effective stresses and mechanical behavior of unsaturated compacted clays is understood. Thus, the applicability of the effective stress principal to such materials cannot be demonstrated at the present time.

2.4 - Conclusion

From the brief review presented, it is apparent that a generalized stress-strain-time relation for compacted clays, suitable for incorporation in a productive model, is not available at the present time. Development of such a model is described in the following sections.

SECTION 3 - EXPERIMENTAL PROCEDURE

In the portion of the study described herein, two commercially prepared soils, "Edgar Plastic Kaolin" (EPK) and "Grundite" were tested. The properties of these materials and the procedure for confined creep testing are described in detail in a previous report (Perloff, 1966). In the series of tests discussed herein confining pressures were generally limited to zero, one and two kilograms per square centimeter, since elastic theory indicates that the maximum horizontal stress on an earth embankment fifty feet high resting on a foundation of the same material is about 1.6 kilograms per square centimeter (Perloff et al, 1967).

Most specifications for field compaction procedures require that the moisture content of the soil should be within a few per cent of optimum moisture content. Also the moisture content is likely to remain sensibly near or slightly lower than the compacted moisture content in a highway embankment, subject to environmental conditions of a particular site. Hence the laboratory specimens were tested at moisture contents at or slightly on the dry side of optimum. One series of tests on Edgar Plastic Kaolin compacted by kneading was conducted at a moisture content far below the optimum moisture content so as to get a clearer picture of the behavior of compacted soils on the dry side of optimum. The failure strength of each sample was determined by the triaxial constant strain rate test, at the end of creep testing.

SECTION 4 - RESULTS AND DISCUSSION

4.1 - Creep Deformations.

Time-dependent deformation in soils is possible under a wide variety of conditions. The published results indicate that this can occur when the soil sample is tested drained, undrained or partially drained; when the sample is normally consolidated or over-consolidated; when it is subjected to a combination of compressive and shearing stresses. Creep itself has been discussed in terms of shearing strain, volumetric strain or an increase in pore water pressure. The term "creep" has been used by different people to mean different phenomena. Reiner (1954) has defined creep as a time-dependent deformation which proceeds at a slow rate. Wu (1966) states that creep is the increase of strain with time. In soil mechanics usage, terms like primary consolidation and secondary compression are generally associated with time-dependent deformation of saturated cohesive soils and the interrelation between these and creep deformation is not yet clearly defined. In the following discussion an attempt is made to define clearly the boundaries of the present investigations with regard to the time-dependent deformation of compacted cohesive soils, and the terminology applied.

In conventional elastic theory it is assumed that only normal stresses contribute to the volumetric strain of an element and the shear stresses have no effect on volume. But one of the most significant factors in the behavior of soils under stress is their volumetric change during shearing, commonly referred to as dilatancy. The failure strength of soils depends to a great extent on their dilatancy characteristics, whether it is in a dry or in a saturated condition. Numerous methods have been proposed to account for this effect in sands and saturated cohesive soils but these are not very useful when the time-dependent stress strain behavior of soils have to be characterized. Application of these approaches to the prediction of response of compacted cohesive soils under load has not yet been attempted. Hence it is proposed to consider compacted cohesive soil as a homogeneous, isotropic and non-dilatant material, unless otherwise stated, in subsequent discussions. The implications of this assumption are discussed below.

The stresses at a point can be thought of, as shown with principal stresses in Figure 4.1.1, as the superposition of the dilatational stress

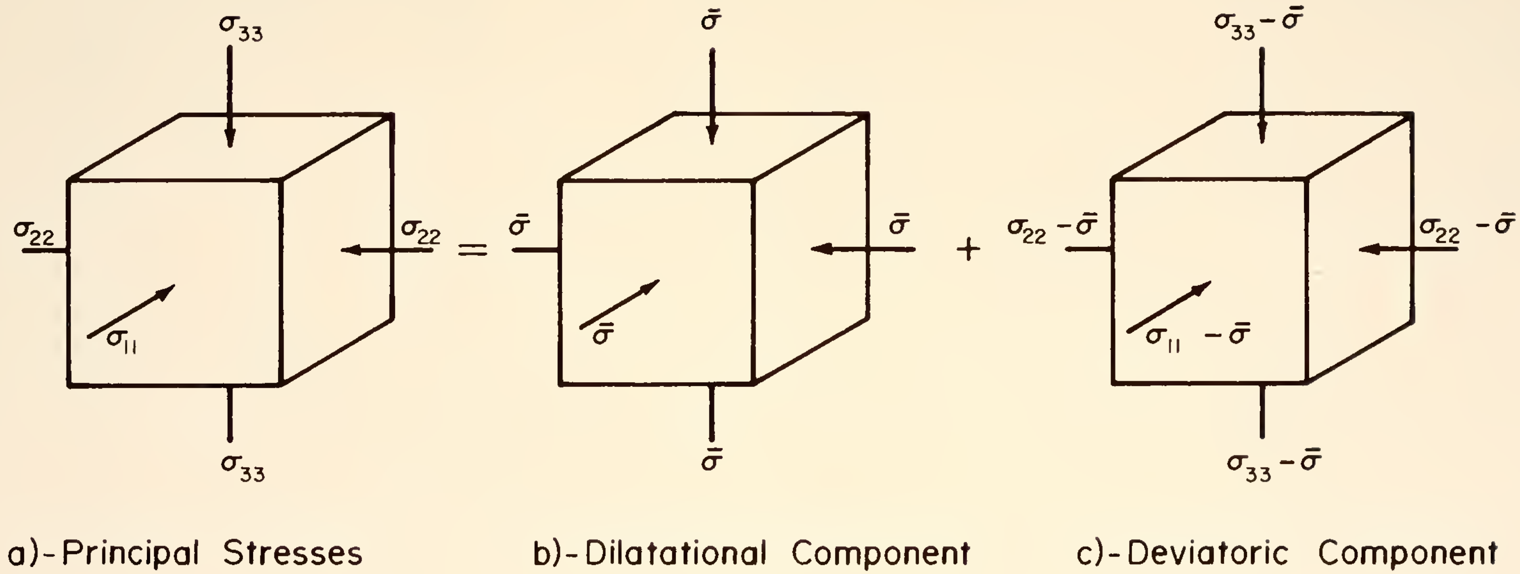


Figure 4.1.1 - Decomposition of The State of Stress at a Point into Dilatational and Deviatoric Components

$\bar{\sigma}$ and the distortional stress $\sigma_{11} - \bar{\sigma}$, $\sigma_{22} - \bar{\sigma}$, $\sigma_{33} - \bar{\sigma}$. As the sum of the normal stresses in the latter is equal to zero, these stresses produce no volume change if the material is isotropic and non-dilatant; they produce only distortion. These stresses are sometimes called deviatoric stresses, since these represent the deviation from a pure dilatational state of stress. So stress and strain at a point can be thought of as being the sum of a pure volume change with no distortion of shape and a pure distortion with no change in volume. We can write the stress and strain system as follows:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \bar{\sigma} & 0 & 0 \\ 0 & \bar{\sigma} & 0 \\ 0 & 0 & \bar{\sigma} \end{bmatrix} + \begin{bmatrix} \sigma_{11}-\bar{\sigma} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\bar{\sigma} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\bar{\sigma} \end{bmatrix} \tag{4.1.1}$$

$$\begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \bar{\varepsilon} & 0 & 0 \\ 0 & \bar{\varepsilon} & 0 \\ 0 & 0 & \bar{\varepsilon} \end{bmatrix} + \begin{bmatrix} \varepsilon_{11}-\bar{\varepsilon} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22}-\bar{\varepsilon} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}-\bar{\varepsilon} \end{bmatrix} \tag{4.1.2}$$

where

$$\bar{\sigma} = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33})$$
$$\bar{\varepsilon} = \frac{1}{3} (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})$$

Two common tests that are generally performed to determine the time dependent characteristics of soil are the one-dimensional consolidation test and the axial creep test. In one-dimensional consolidation test we measure the axial deformation, which is the response due to an applied axial load. Since the soil sample is laterally confined, there is no lateral deformation. Generally it is not possible to have an idea of the lateral stresses imposed by the restraint. Hence the stress system is not fully defined. Also the axial strain is equal to the volumetric strain since the lateral strains are zero. The stress system is three dimensional while the strain system is one dimensional. They can be written, at a particular time (t), as:

$$\begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \bar{\sigma} & 0 & 0 \\ 0 & \bar{\sigma} & 0 \\ 0 & 0 & \bar{\sigma} \end{bmatrix} + \begin{bmatrix} \sigma_{11}-\bar{\sigma} & 0 & 0 \\ 0 & \sigma_{22}-\bar{\sigma} & 0 \\ 0 & 0 & \sigma_{33}-\bar{\sigma} \end{bmatrix} \tag{4.1.3}$$

$$\varepsilon_{11} = \frac{1}{3}\varepsilon_{11} + \frac{2}{3}\varepsilon_{11} \tag{4.1.4}$$

Since the axial strain is equal to both the volumetric strain and distortional strain, it is not possible to separate them individually to study their behavior.

In a triaxial creep test the specimen is consolidated to the required pressure by the application of confining pressure. Generally the resulting volumetric strain is not measured. After the consolidation a deviator stress $(\sigma_1-\sigma_3)$ is applied as a step load and the resulting axial strain is measured. In this test the stress system is fully defined while the strain system is not fully defined. We can write them at a particular time (t) as:

$$\begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \bar{\sigma} & 0 & 0 \\ 0 & \bar{\sigma} & 0 \\ 0 & 0 & \bar{\sigma} \end{bmatrix} + \begin{bmatrix} \sigma_{11}-\bar{\sigma} & 0 & 0 \\ 0 & \sigma_{22}-\bar{\sigma} & 0 \\ 0 & 0 & \sigma_{33}-\bar{\sigma} \end{bmatrix} \tag{4.1.5}$$

$$\begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \bar{\varepsilon} & 0 & 0 \\ 0 & \bar{\varepsilon} & 0 \\ 0 & 0 & \bar{\varepsilon} \end{bmatrix} + \begin{bmatrix} \varepsilon_{11}-\bar{\varepsilon} & 0 & 0 \\ 0 & \varepsilon_{22}-\bar{\varepsilon} & 0 \\ 0 & 0 & \varepsilon_{33}-\bar{\varepsilon} \end{bmatrix} . \tag{4.1.6}$$

In this case it is not possible to separate out the individual strains as we do not know the magnitude of ε_{22}, ε_{33} and hence $\bar{\varepsilon}$.

It is obvious that the above commonly performed tests are inadequate to obtain reliable estimates of the general time-dependent material properties of the soil specimen tested since the stress and strain systems are not fully defined. The stress and strain systems must be fully defined in order to get the necessary moduli for the soil tested. Hence it was decided to measure the lateral deformation in the triaxial creep test to determine the system of strains. The lateral strain can be taken to be equal to both

ε_{22} and ε_{33}. By calculating the difference between the axial strain and lateral strain we obtain the maximum distortional strain, which is the response due to the application of the deviator stress.

4.2 - Characterization of Time-Dependent Nature of the Soils Tested.

In Figure 4.2.1 the maximum shear strain obtained during a typical triaxial compression creep test on an impact compacted specimen is plotted against time for various stress levels. For the duration of the test period the creep curves appear as straight lines in a logarithmic plot. Thus the relation between maximum shear (distortional) strain and time can be expressed as,

$$(\varepsilon_1 - \varepsilon_3)(t) = c\ t^n \tag{4.2.1}$$

where

$(\varepsilon_1 - \varepsilon_3)(t)$	=	maximum shear strain at time t;
$\varepsilon_1(t)$	=	axial strain at time t;
$\varepsilon_3(t)$	=	radial strain at time t;
t	=	time elapsed after the application of load;
c, n	=	constants;

As discussed in a preceding report (Perloff, 1966), this relation appeared to be valid whether the sample was tested drained or undrained; whether it was the first loading or subsequent loadings, whether the sample was confined or unconfined; for all stress levels, for all moisture contents, and for all types of compaction that were being used in this study. These results are in general agreement with those obtained for uniaxial measurements, by Singh and Mitchell (1968, 1969), Murayama and Shibata (1961), and Lara-Tomas (1962). Equation 4.2.1 implies that the shear creep compliance can be expressed as

$$J(t) = \Delta J(t) \tag{4.2.2}$$

instead of the usual

$$J(t) = J(0) + \Delta J(t) \tag{4.2.3}$$

where

$J(0)$ = initial value of compliance (t=0),

$\Delta J(t) = J(t) - J(0)$ = transient component of compliance.

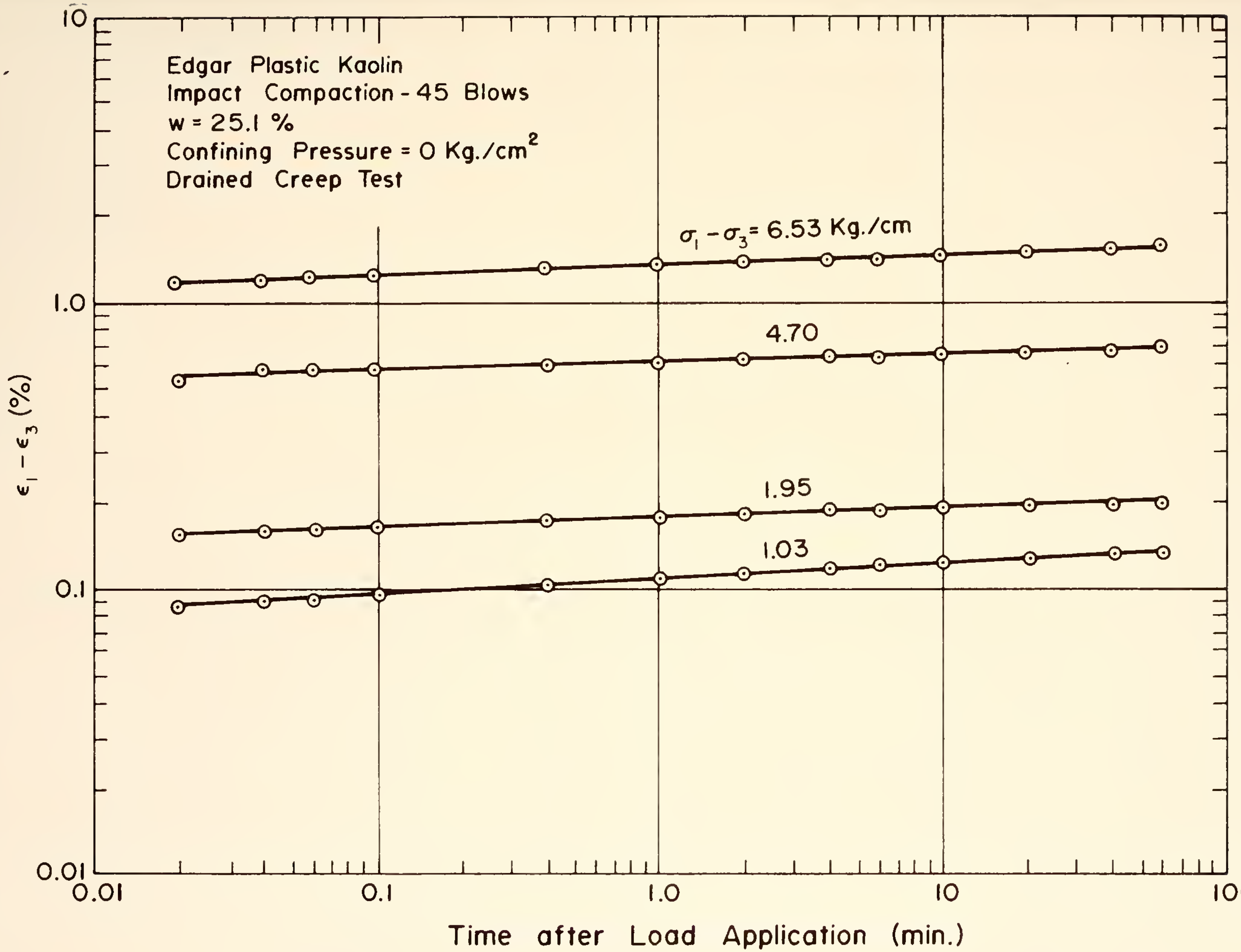


Figure 4.2.1 - Relation Between Maximum Shear Strain and Time

This form is commonly applied in characterizing polymeric materials.

To judge the statistical reliability of Equation 4.2.1, a least squares analysis was performed for some of the experimental data. It was found that the correlation coefficient in all the cases analyzed was greater than 0.99, and examination of residuals for the above cases indicated that the assumptions made for the analysis were valid. Hence it was concluded that Equation 4.2.2 was a valid representation of the shear creep compliance in the tests described.

A complete material characterization can be given for a homogeneous, isotropic, non-dilatant material by describing one other material parameter. For a triaxial compression creep test in which the lateral pressure remains fixed, the negative of the ratio between lateral strain and axial strain at any time can be usefully thought of as the equivalent Poisson's ratio at that time. This ratio constitutes the second parameter required.

The relation between Poisson's ratio and time for various stress levels is shown for a typical test in Figure 4.2.2. At lower stress levels the value of Poisson's ratio decreased with increase in time; but as the shear stress level increased, the value of Poisson's ratio increased with time. This trend appeared to be true for all the tests that were conducted during this study, irrespective of test conditions. It is to be noted that the increase or decrease in value at a particular stress level with time was comparatively small. Hence it is believed reasonable to assume that Poisson's ratio is essentially independent of time.

4.3 - Effect of Loading History on Creep Behavior.

In a previous report, (Perloff, 1966) it was shown that the shear strain induced by the load application is substantially larger for the initial loading than for the succeeding cycles, even though the form of the straight line logarithmic relationship is valid for all the cycles. Also the response in each of the subsequent cycles after the initial loading cycle was found to be essentially the same. A similar result was observed by Lara-Tomas (1962). This result appeared applicable irrespective of the duration of the first loading cycle and the length of the unloading period between load cycles.

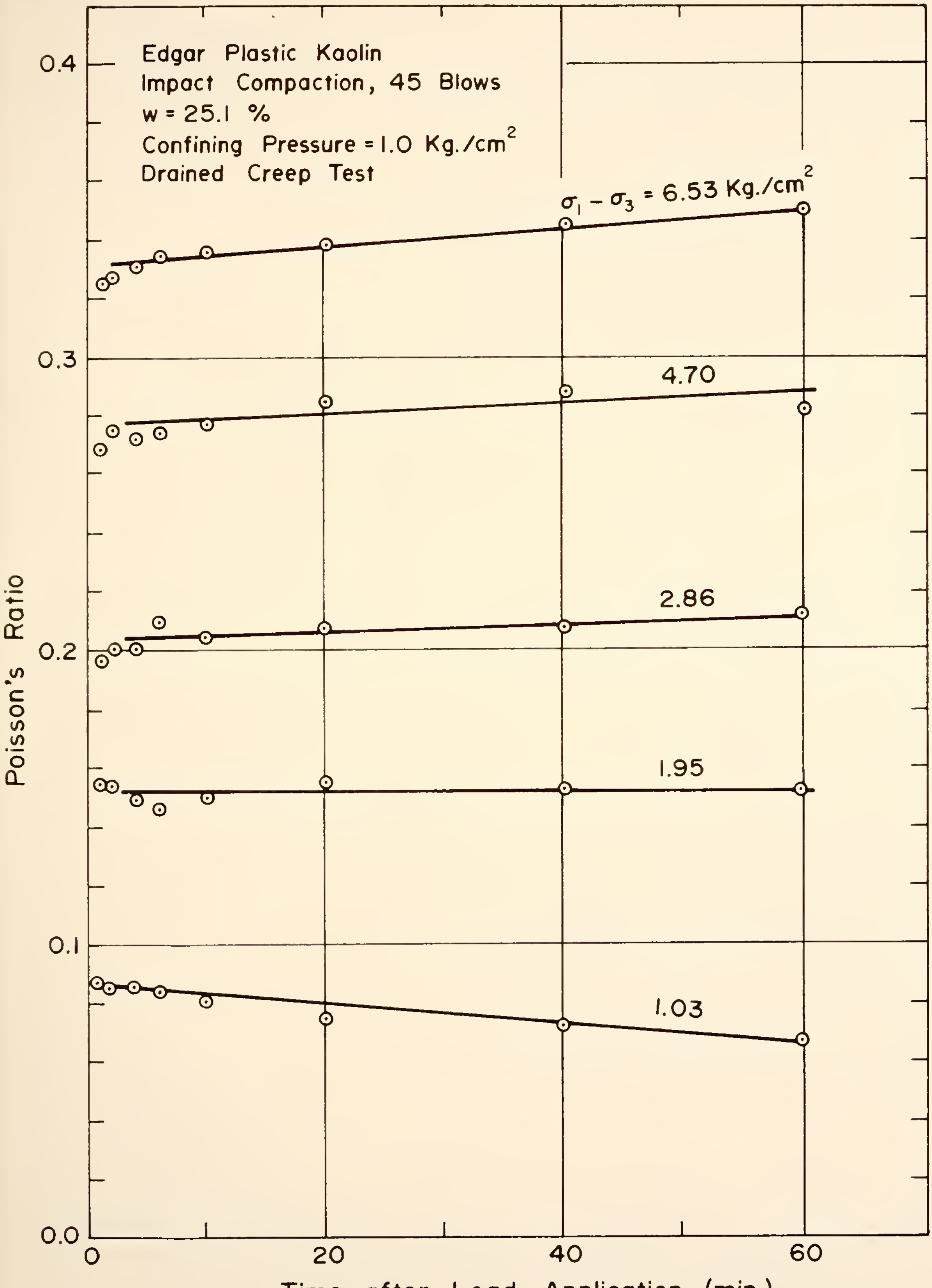


Figure 4.2.2 - Relation Between Poisson's Ratio and Time

It was not clear, however, whether the above conclusions were valid when the initial load was applied for only a very short duration of time. Hence triaxial creep tests were performed wherein the duration of the first cycle of loading was limited to one minute. The specimen was then allowed to relax for fifteen minutes and then reloaded. Figures 4.3.1 and 4.3.2 show the results of tests on two "identical" specimens, one of which was subjected to the short first load cycle described above, the other of which experienced the conventional one-hour first load followed by a one-half hour unload period. Other conditions of testing for the two samples were identical. The figures show the second cycle responses for the two specimens. It is seen that the responses are similar and the difference between these two cases is small enough to be attributed to sample-to-sample variation. Hence it can be concluded that the response in second and subsequent cycles do not depend on the duration of the first loading cycle.

In order to predict the deformation of earth embankments due to their own weight, a decision has to be made whether to use the response due to first or subsequent cycles of loading to characterize the material. Consideration of the compaction procedure in the field is helpful in this regard.

Consider, for example, an earth embankment of fifty feet in height. Assuming the unit weight of soil mass as 130 PCF, the maximum vertical stress will be approximately 45 psi. When the soil is compacted in the field, the contact pressure will be approximately 250 psi under a sheeps foot roller. During the compaction of subsequent lifts, the pressure will be less, but for the lift immediately above, it will be higher than the stresses that will be induced by the self-weight of the mass. This implies that the compacted layer is subjected to a number of cycles of greater magnitude than the subsequent dead load stress that will be imposed. Hence it would seem reasonable to use the response due to second and subsequent loadings in the analysis. Consequently all further results will be reported for second or subsequent cycles of loading.

It was previously reported (Perloff, 1966) that the results for the second and subsequent cycles of load of tests in which the soil had been subjected to a series of loads were essentially similar to those from the tests in which each load was applied to a separate soil specimen. Hence, for the tests reported herein, a single specimen was used for each load series.

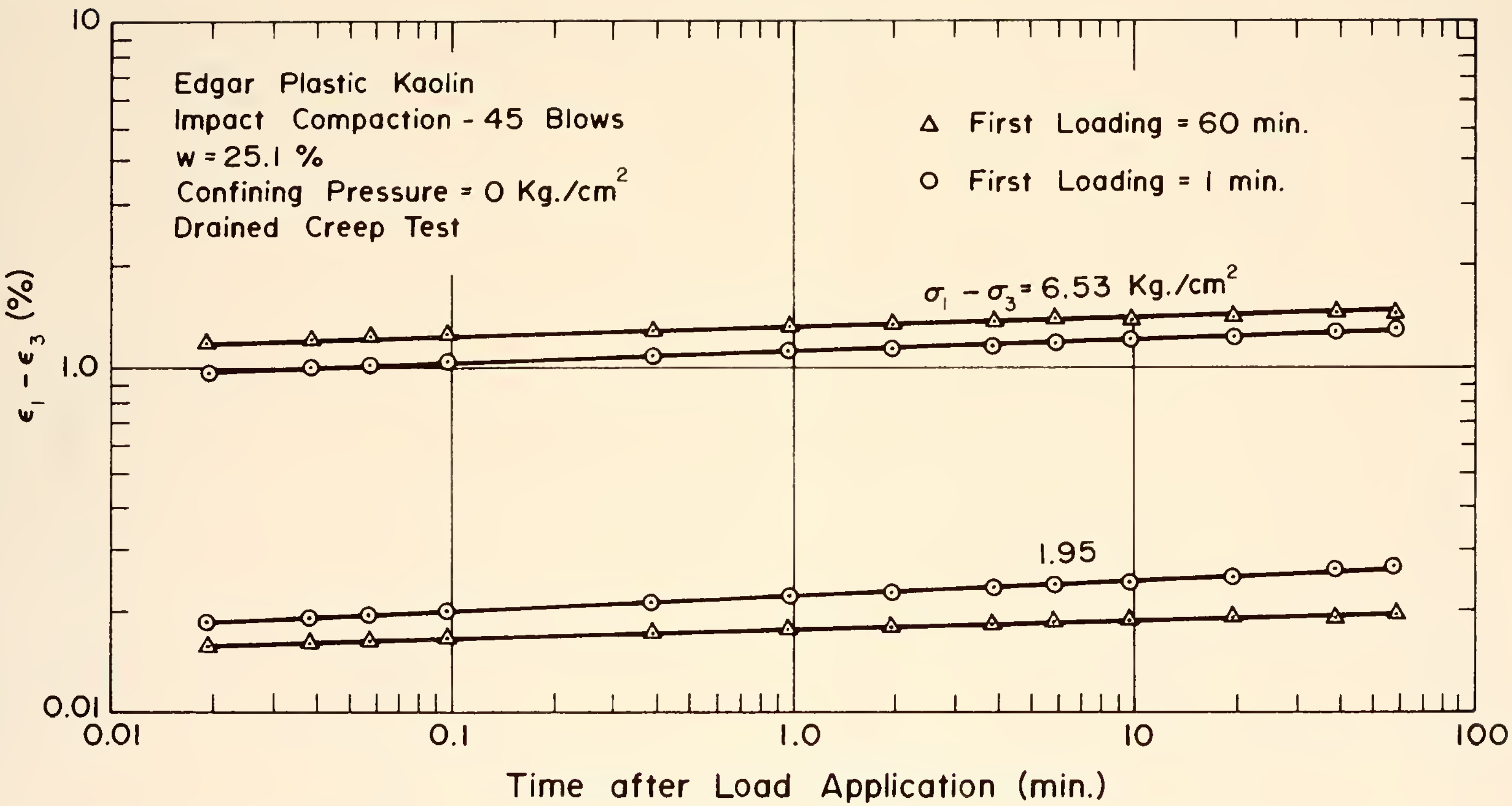


Figure 4.3.1 - Comparison of Relation Between Maximum Shear Strain and Time During Second Load Cycle, for First Loading Periods of one Minute and one Hour

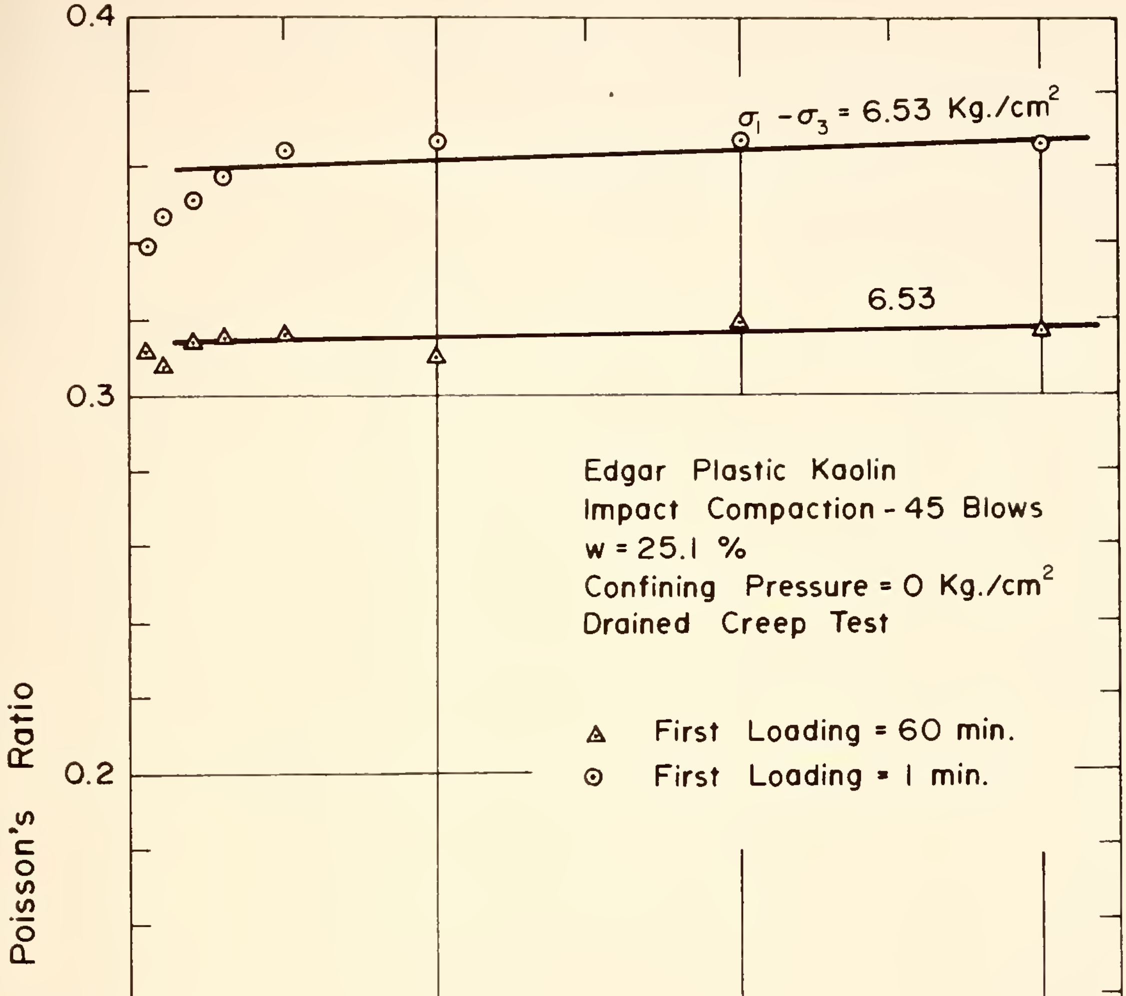


Figure 4.3.2 - Comparison of Relation Between Poisson's Ratio and Time During Second Load Cycle, for First Loading Periods of one Minute and one Hour

4.4 - Effect of Stress Level on Creep Behavior.

The effect of shear stress level at a particular confining pressure on c and n (Equation 4.2.1), and Poisson's ratio is shown in Figures 4.4.1 and 4.4.2. The parameters c and n were calculated for each creep test by using a least square approximation of polynomial curve fitting for the data, using computer programs available at the Purdue Computing Center. Since Poisson's ratio was nearly independent of time, its magnitude at one minute was used in these and subsequent figures.

It is seen in Figures 4.4.1 and 4.4.2 that at low stress levels c can be considered to be a linear function of principal stress difference; but as the stress increases, c increases nonlinearly. Thus, over the majority of the practical range of shear stresses, the mechanical response of compacted cohesive soil is nonlinear.

As reported previously (Perloff, 1966), the magnitude of n was independent of the stress level for a particular specimen. There was some variation from sample to sample; the value of n varied from 0.025 to 0.058 for kaolinite and from 0.054 to 0.078 for grundite. Thus n was assumed constant for a particular soil type, compaction type, and moisture content, irrespective of magnitude of principal stress difference and confining pressure.

For a particular confining pressure there was an increase in the magnitude of Poisson's ratio as the principal stress difference increased. The plots indicated that there was an interaction between the confining pressure and stress level on the magnitude of Poisson's ratio. The value of Poisson's ratio varied from 0.12 to 0.46 for kaolinite and from 0.13 to 0.38 for grundite. In most of the published literature the magnitude of Poisson's ratio has generally been assumed to be near 0.5 irrespective of applied stress level.

To verify the above results, constant strain-rate triaxial compression tests with lateral strain measurements were performed on a specimen that had already undergone creep testing. The equivalent Poisson's ratio, calculated at various stages of the test, is shown in Figure 4.4.3. Also shown are the results for the creep test performed prior to constant strain-rate testing. The good agreement is evident.

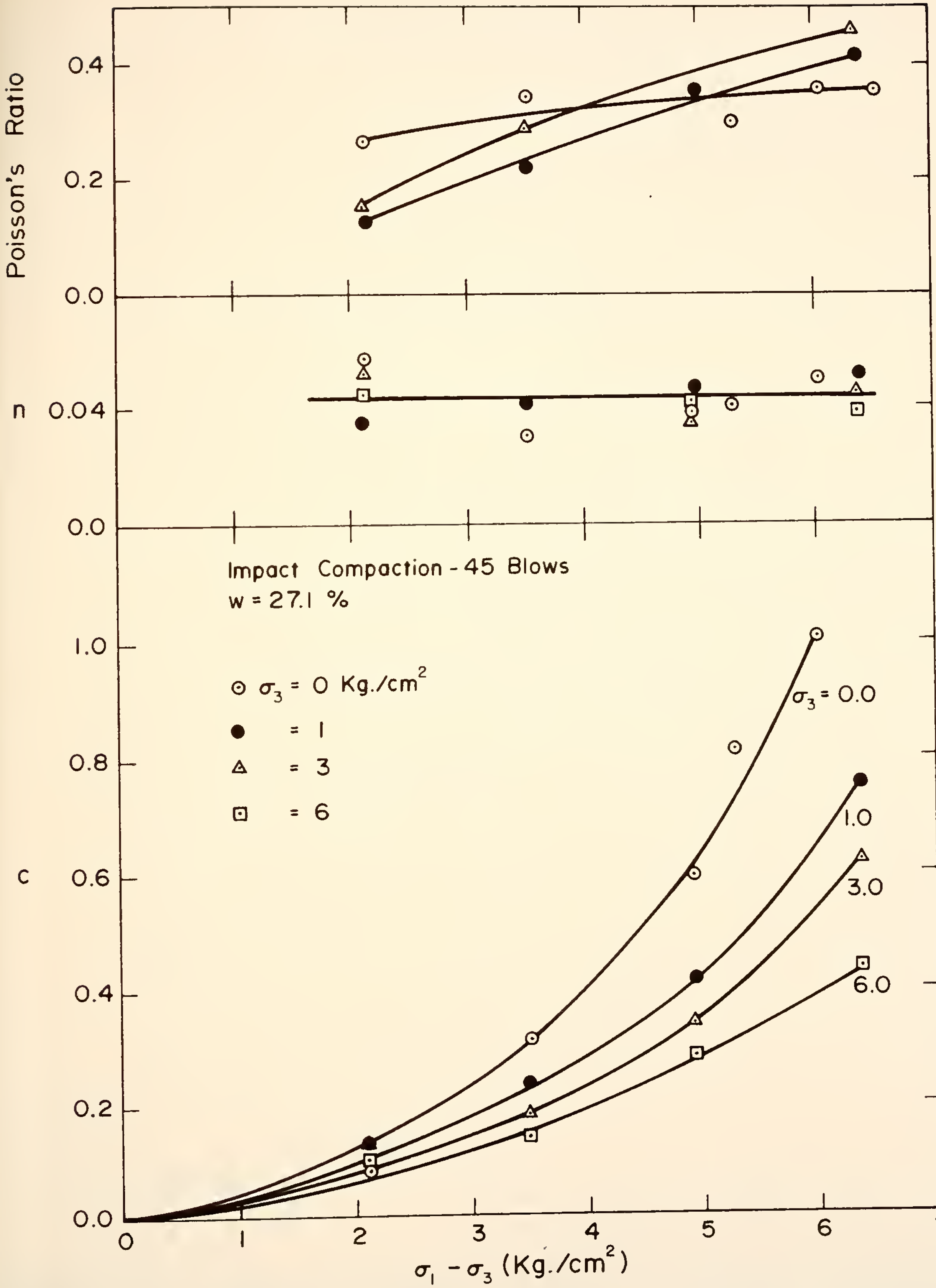


Figure 4.4.1 - Effect of Stress Levels on Creep Parmeters for EPK

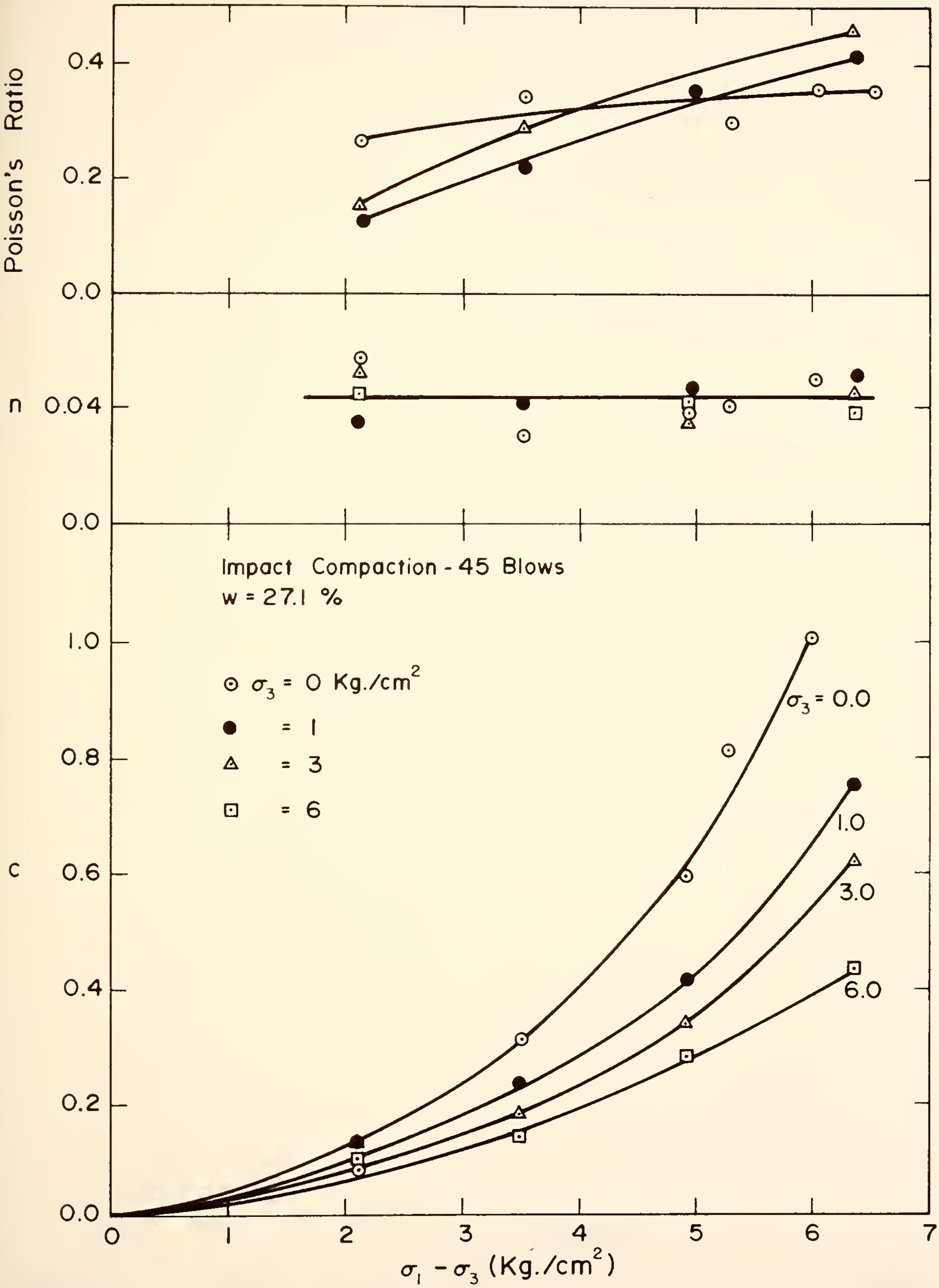


Figure 4.4.1 - Effect of Stress Levels on Creep Parmeters for EPK

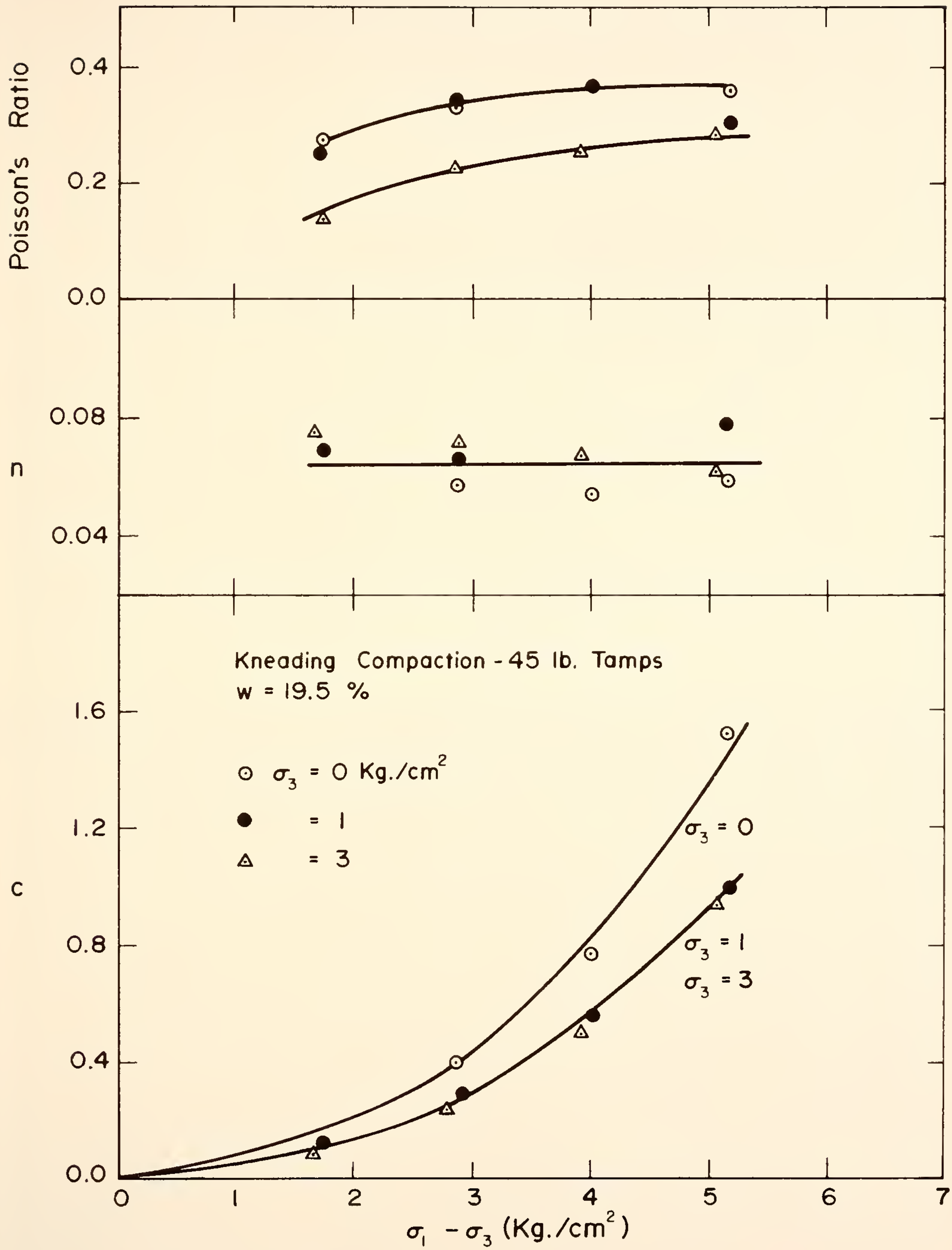


Figure 4.4.2 - Effect of Stress Level on Creep Parameters for Grundite

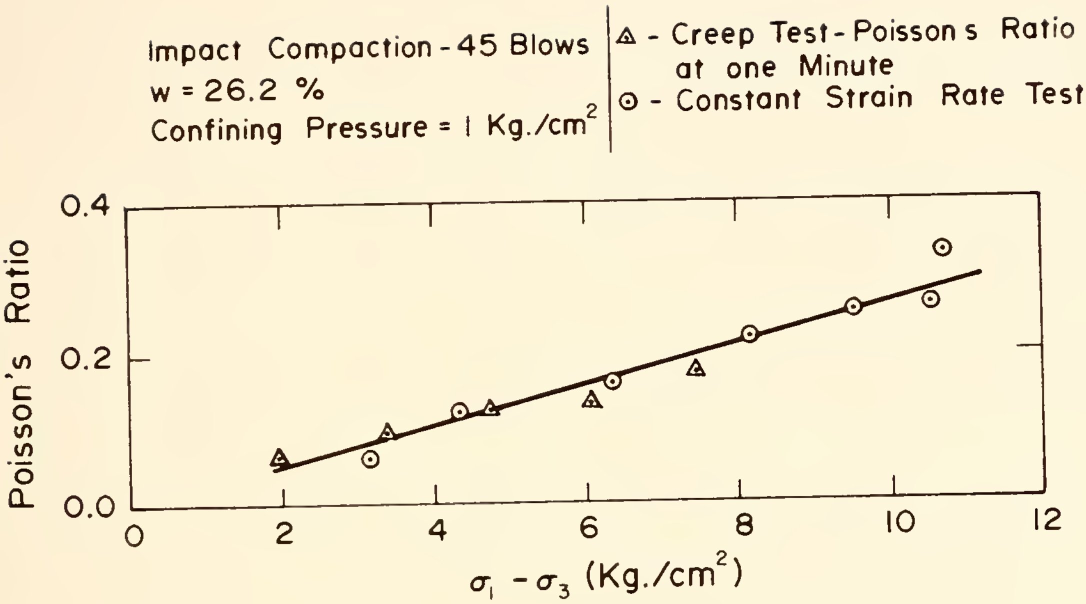


Figure 4.4.3 - Comparison Between Constant Strain Rate and Creep Tests - Effect of Stress Level on Poisson's Ratio for E P K

4.5 - Effect of Confining Pressure on Creep Behavior.

The effect of confining pressure on the parameters c, n and Poisson's ratio is shown in Figures 4.5.1 and 4.5.2. The data for these figures are the same as for Figures 4.4.1 and 4.4.2; but they are replotted to indicate the influence of confining pressure. The data indicate that the confining pressure does not affect the magnitude of c markedly at low principal stress differences. But as stress levels increase, the reduction in c produced by increased confinement also becomes more significant. This is particularly noticeable when the confining pressure is increased from zero to one kilogram per square centimeter. This effect is similar to the greater increase in peak strength when the confining pressure is increased from zero to one kg/cm^2 than when the confining pressure is increased above one kg/cm^2 (Figure 4.5.3).

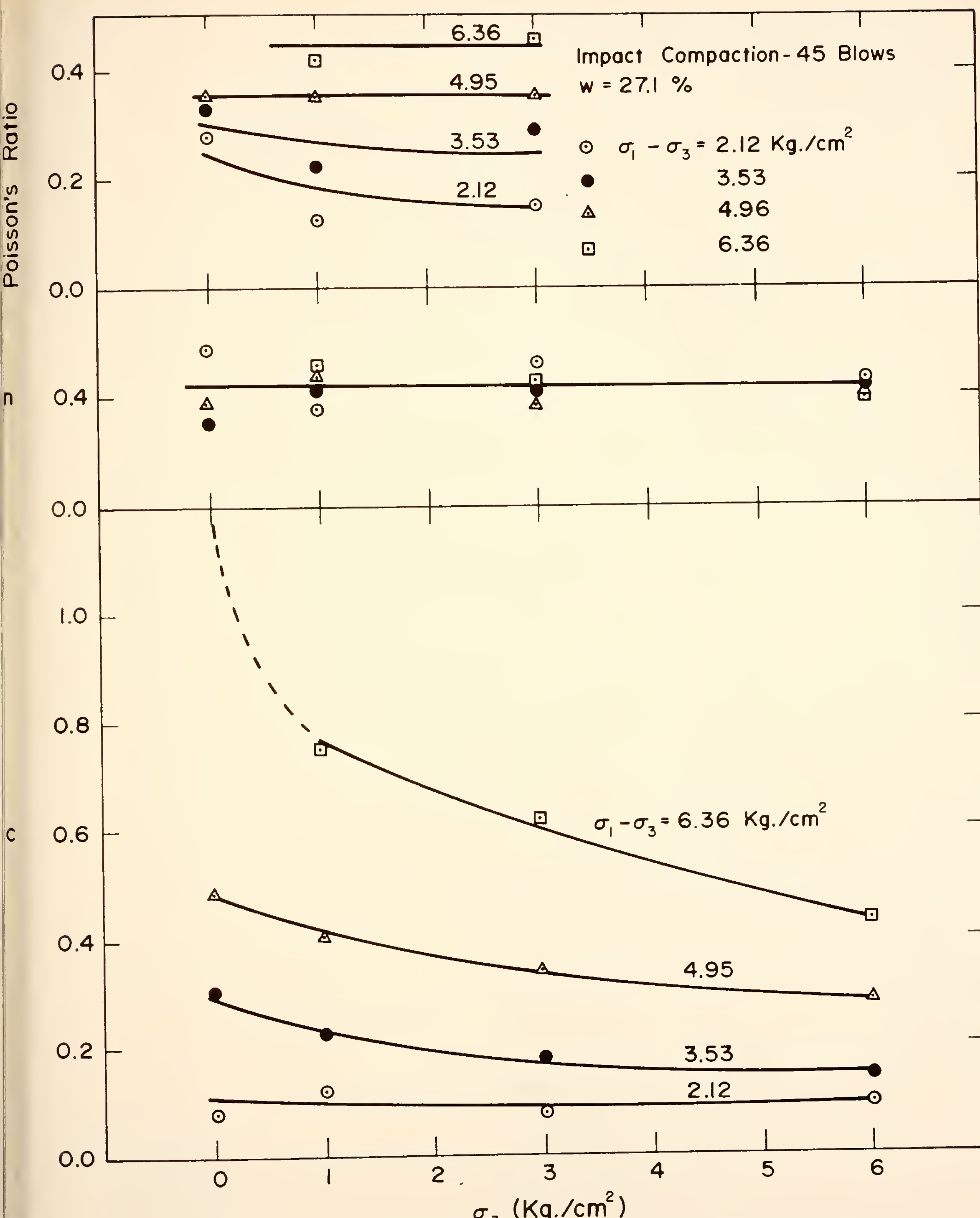
No consistent relationship between n and the confining pressure was found, suggesting that for all practical purposes n is independent of confining pressure.

At low stress levels Poisson's ratio decreased with increasing confining pressures, while at higher stress levels it was almost independent of confinement.

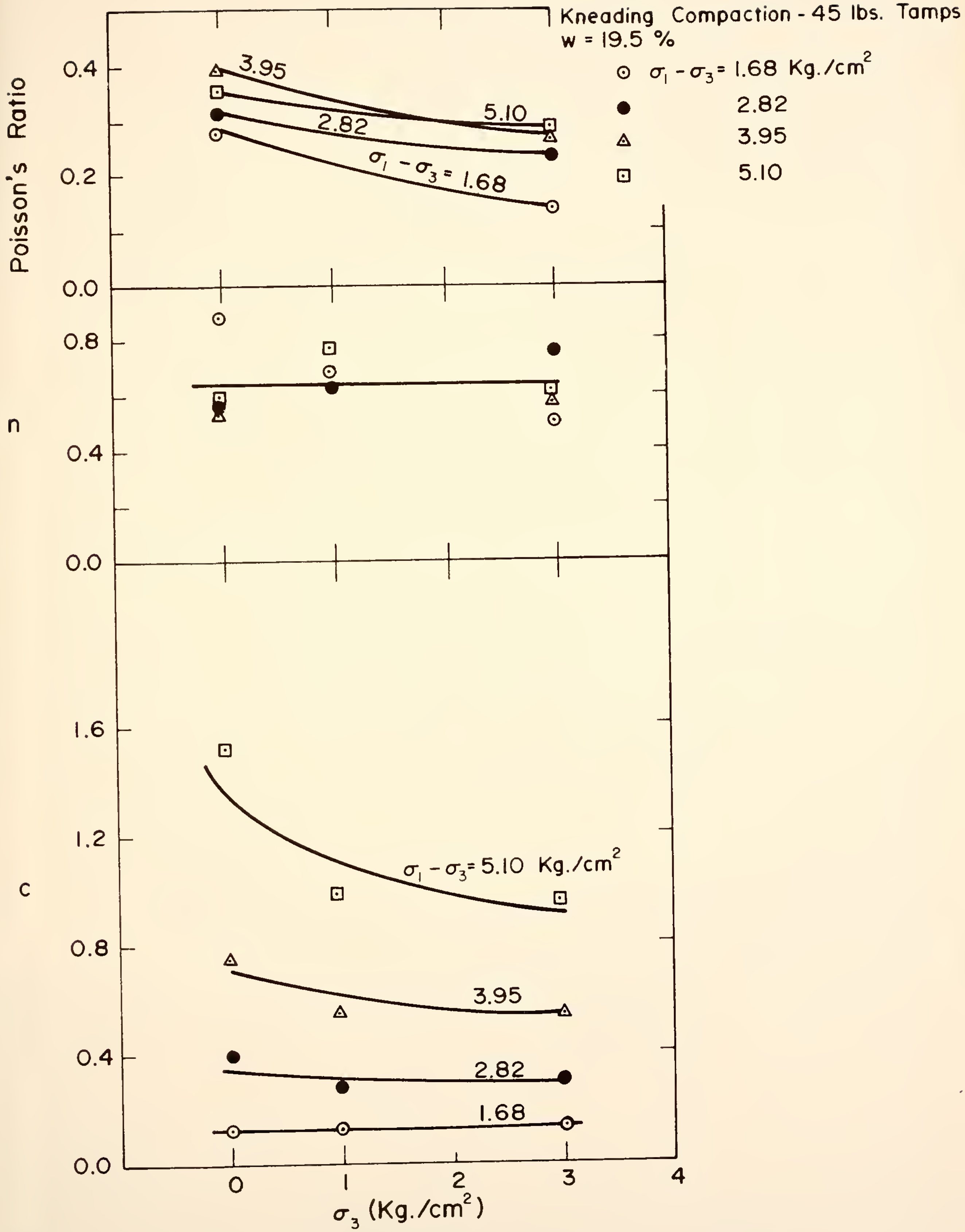
4.6 - Effect of Moisture Content and Compactive Effort on Creep Behavior.

To determine the effects of moisture content on the parameters c, n and Poisson's ratio, compacted samples were tested at different moisture contents: on the dry side of optimum moisture content, at optimum, and on the wet side of optimum. Two different compactive efforts, 45 and 120 blows per sample, were used in specimen preparation. The soil used was grundite.

The creep test results shown in Figures 4.6.1 and 4.6.2 indicate that when the soil is compacted dry of optimum moisture content its creep response is almost linearly viscoelastic over a wide range of stress. At and above optimum the mechanical behavior was nonlinear even at relatively low stress levels. As moisture content was increased, the magnitude of c at a particular stress level also increased. Similar behavior was observed for both compactive efforts.



ure 4.5.1 - Effect of Confining Pressure on Creep Parameters for EPK



igure 4.5.2 - Effects of Confining Pressure on Creep Parameters for Grundite

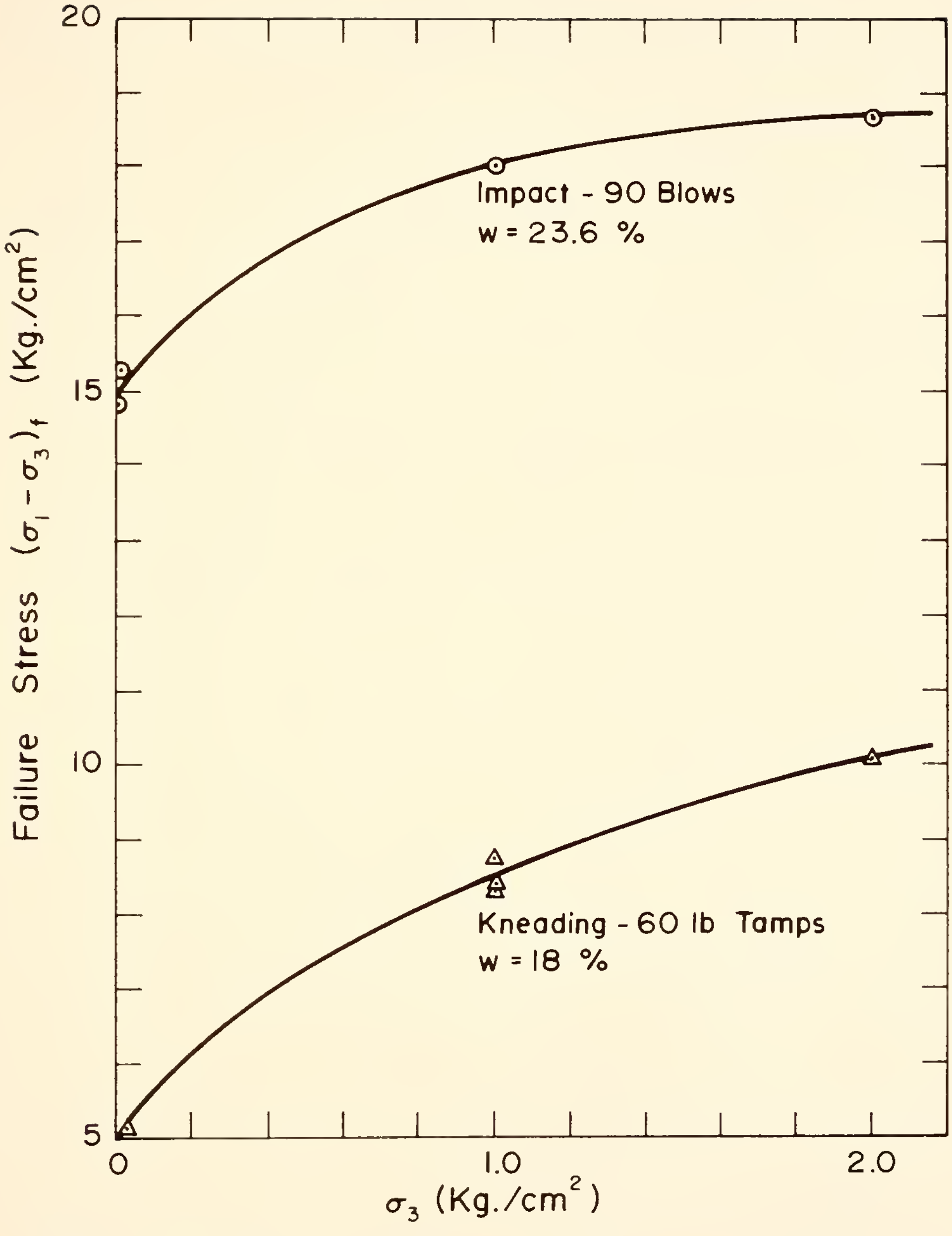


Figure 4.5.3 - Effect of Confining Pressure on $(\sigma_1 - \sigma_3)_f$ for EPK

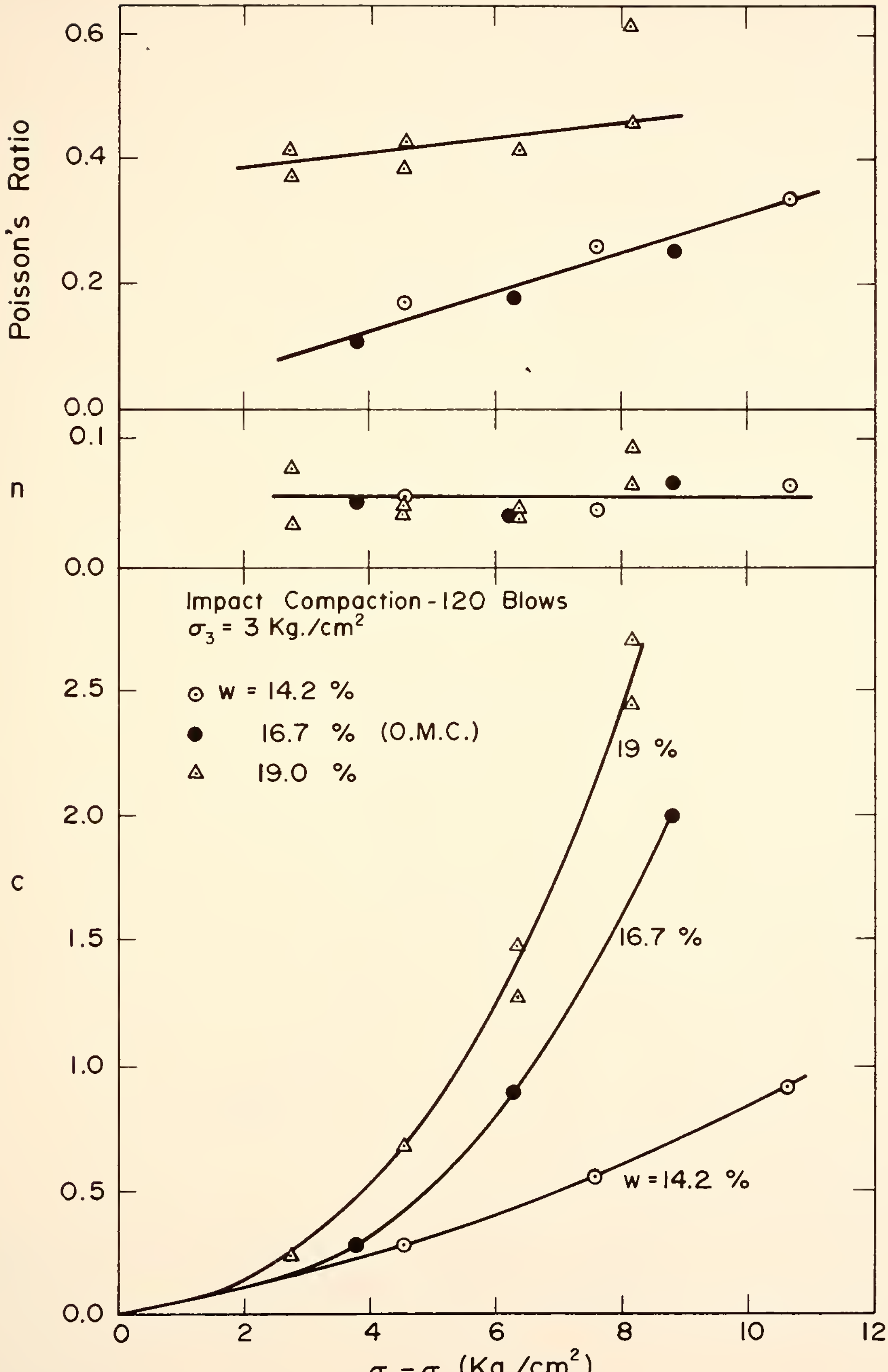


Figure 4.6.1 - Effect of Moisture Content on Creep Parameters for Grundite

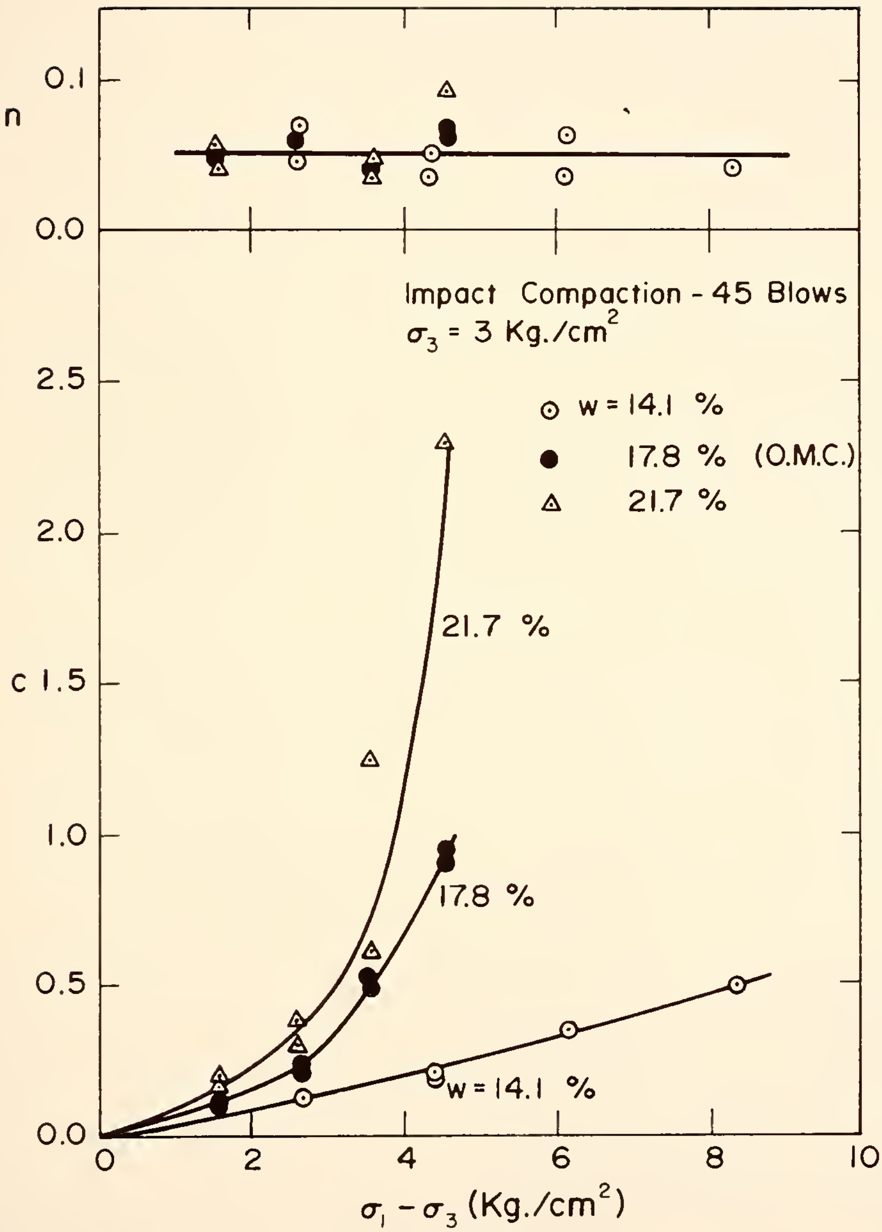


Figure 4.6.2 - Effect of Moisture Content on Creep Parameters for Grundite

The factor n was essentially independent of moisture content and compactive effort. The magnitude of n varied from 0.03 to 0.09 for both types of compaction. The results obtained by Singh and Mitchell (1968) indicate that n for saturated cohesive soils vary from 0 to 0.25 for secondary consolidation, while the results of Lara-Tomas (1962) suggests that n will be about 0.20 for hollow cylindrical compacted cohesive samples prepared by kneading compaction. Hence there seems to be a wide variation in the value of n determined by different tests, for different soils, prepared by different types of compaction efforts. Yet the results presented herein show that for a given soil and compaction type, n can be taken as constant.

The interpretation of the influence of moisture content on Poisson's ratio are based on one series of tests only. The magnitude of Poisson's ratio was appreciably the same at moisture contents below and at the optimum. But at moisture contents greater than the optimum moisture content there was a substantial increase in its value. This is likely due to the fact that at higher water contents the degree of saturation is then close to 100 per cent.

4.7 - Correlation of Creep Response to Failure Strength

It was shown previously (Perloff, 1966) that creep response, as characterized by the parameter c, could be described usefully as a function of the stress ratio,

$$\frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f}$$

When this was done, it was found that the effects of water content and confining pressure were approximately accounted for.

This observation was verified and extended to a quantitative statement by the present investigation. For this purpose, a series of tests was performed on EPK in which failure strength was measured for each specimen after creep testing. The levels of various factors used in the analysis are shown in Table 4.7.1.

Table 4.7.1 - Levels of Various Factors in the Analysis

Type of Compaction	$\frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f}$	σ_3 kg. per sq. cm.	Moisture Content
Kneading	0.1 to 0.9	0, 1, 2	17.6 to 27.5
Impact	0.1 to 0.7	0, 1	23.8 to 25.1

A polynomial regression curve of the form

$$c = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{11} x_1^2 + \beta_{33} x_3^2 + \beta_{111} x_1^3$$

$$+ \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \varepsilon \qquad (4.7.1)$$

in which

$$x_1 = \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f} \qquad (4.7.2)$$

$$x_2 = \sigma_3 \qquad (4.7.3)$$

$$x_3 = w \qquad (4.7.4)$$

was obtained for the two types of compaction, using the "Weighted Regression Analysis Program" available at Purdue University Computing Center. The probability level for elimination of variables was chosen as 0.90.

For impact compaction it was found that c could be written as

$$c = \beta_o + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{111} x_1^3 \qquad (4.7.5)$$

or

$$c = \beta_o + \beta_2 \sigma_3 + \beta_3 w + \beta_{12} \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f} \sigma_3 + \beta_{111} \left[\frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f}\right]^3 \qquad (4.7.6)$$

The correlation coefficient for the data used was found to be 0.993.

This result was compared to a polynomial fit in which only the terms containing x_1 in Equation 4.7.1 were retained. For this case the correlation coefficient was found to be 0.984. Thus, addition of other factors to the stress ratio does not add significantly to the correlation coefficient.

For the kneading compaction, the regression analysis resulted in the following expression:

$$c = \beta_0 + \beta_{11}\left[\frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f}\right]^2 + \beta_{12}\frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f}\sigma_3 + \beta_{13}\frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f} w \quad (4.7.7)$$

The correlation coefficient was 0.985. When a polynomial was fitted using the stress ratio alone, the correlation coefficient was 0.980.

Hence it was concluded that the effects of moisture content and confining pressure on c can be accounted for by expressing it as a function of $\frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f}$ alone. This relation, as well as the effect of stress ratio on n and Poisson's ratio are shown in Figures 4.7.1 and 4.7.2 for various water contents and confining pressures.

From these figures, it appears that the relationship between c and stress ratio, can be considered linear only at relative low stress ratios.

Poisson's ratio increased with increase in stress ratio. The magnitude varied from 0.02 to 0.23 for kneading compaction and from 0.02 to 0.33 for impact compaction. A straight line relation of the form

$$\text{Poisson's ratio} = \beta_0 + \beta_1 \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f} \quad (4.7.8)$$

was fitted to the data. The correlation coefficient was 0.85 for the kneading compaction and 0.91 for the impact compaction. These values are considered satisfactory and a linear relationship between Poisson's ratio and the stress ratio can be assumed with reasonable accuracy up to about 0.8 of the failure stress.

The magnitude of n is essentially independent of the value of stress ratio, even though there are variations between samples.

4.8 - Scaling Effect

The properties of materials as obtained by the laboratory tests cannot generally be applied directly to the analysis of prototype structures. When the specimens are small in comparison with the actual soil mass, the possibility of a scaling effect must be considered.

Deformation of a compacted cohesive soil is a continuous process with time. The main factors that may contribute to controlling the rate of

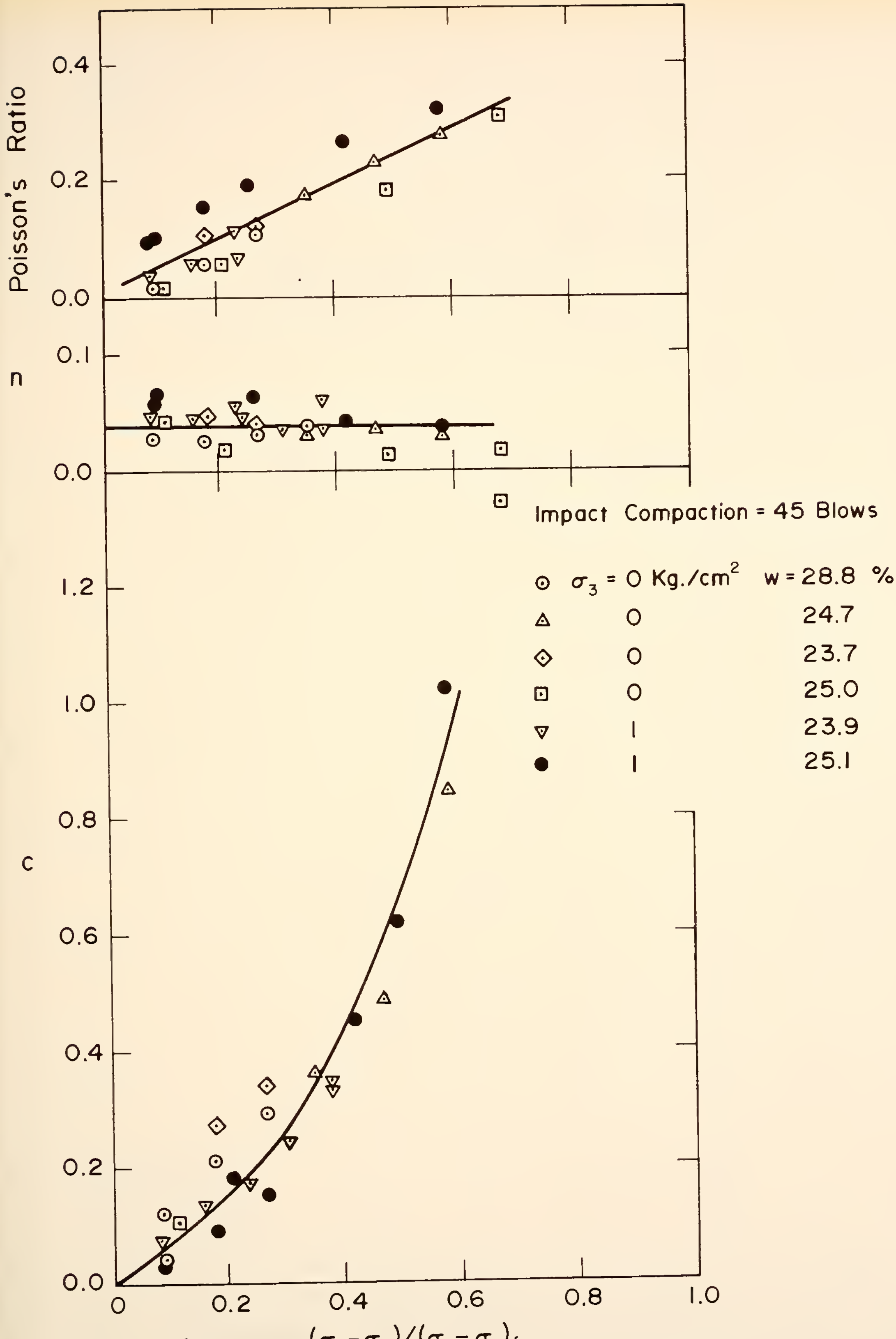


Figure 4.7.1 - Effect of Stress Ratio on Creep Parameters for EPK

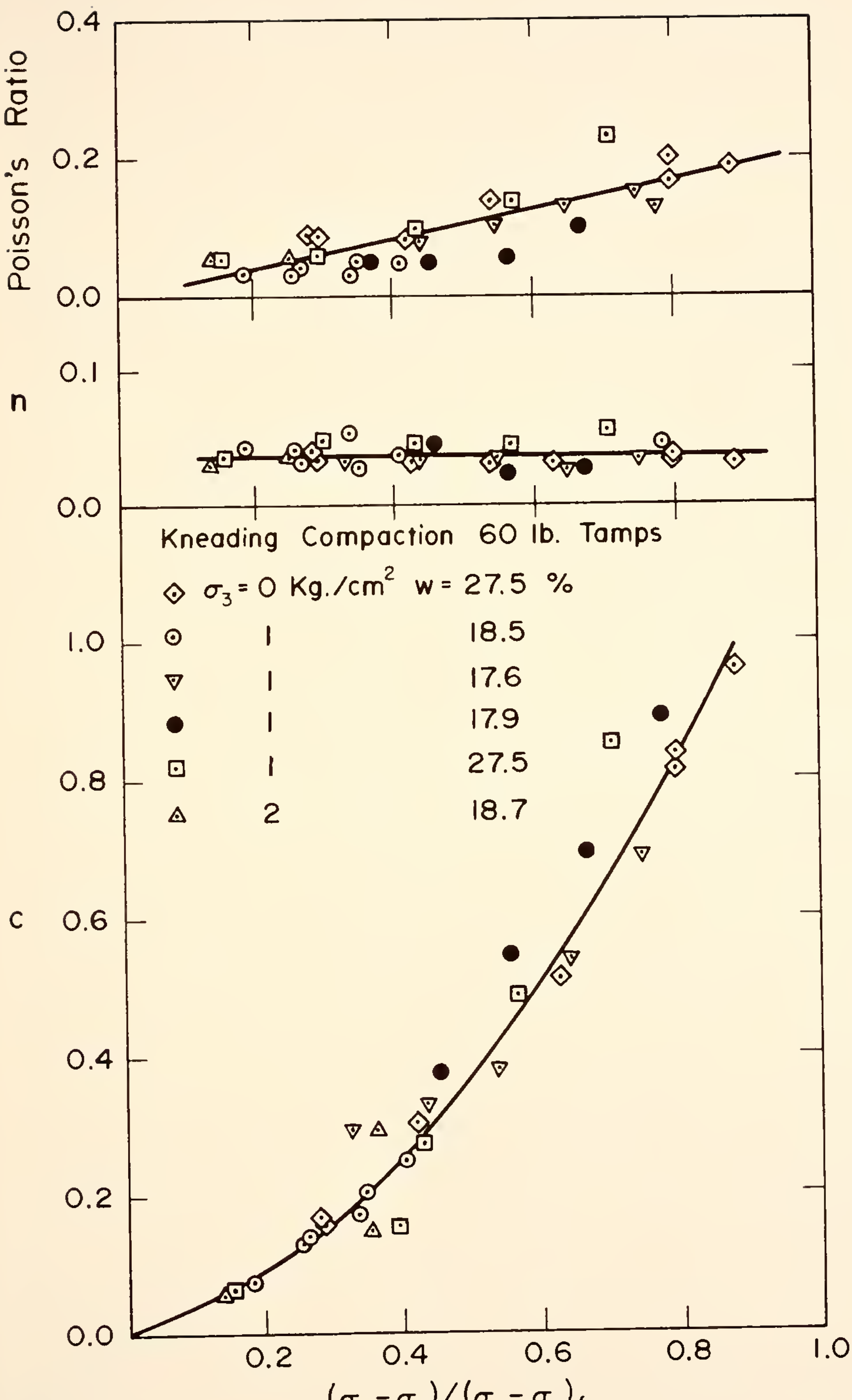


Figure 4.7.2 - Effect of Stress Ratio on Creep Parameters for EPK

time-dependent deformation are:

1. Escape of air through the pores;
2. Compression of air within the pores;
3. Dissolution of air in water;
4. Redistribution of water within the pores; and
5. Deformation of soil structure itself.

Relevant information available on the volume change and shear deformation characteristics of compacted cohesive soils is summarized in the following:

1. There is no expulsion of water from the compacted soil under load, even when the degree of saturation is very high (Yoshimi and Osterberg, 1963).
2. Substantial negative pore pressures exist in the capillaries of the partly saturated soil (Olson and Langfelder, 1965).
3. The negative pore water pressures existing in the capillaries of compacted soil are likely to be greater than the loadings used in the creep deformation studies.
4. When the soil is compacted at optimum or dry of optimum water content, the pores are interconnected and there will be no impediment to free flow of air (Yoshimi and Osterberg, 1963).
5. The rheological characteristics of the soil structure is the most dominant factor influencing the time-dependent shear deformation behavior of compacted cohesive soils.

Immediately after loading there will be excess air and water pressure induced in the partly saturated sample. But since the induced water pressure is generally far less than the negative pore water pressure existing in the soil, there will be no escape of water (Yoshimi and Osterberg, 1963). It is difficult to predict the redistribution of water in the pores due to pressure gradients caused by the loading. However the volume change involved in this process is likely to be negligible. So the increase in pore water pressure due to loading has little influence in the deformation of partly saturated soils.

The distribution of pores will probably be the same irrespective of sample size, since similar methods are used in the compaction processes. Hence for the same increase in pressure, the compression of air in the entrapped voids and the dissolution of air in water will be the same irrespective of the sample size. Hence there will be no scaling effect due to these factors.

The rheological properties of the soil are not likely to alter with increase or decrease in size, since the soil skeleton will possess the same basic properties in both cases. So scale effect in this case can also be neglected.

The excess pore air pressure has to be dissipated in the course of time. The time taken for dissipation is likely to be very small in the laboratory specimen, due to the fact that length of drainage path is small and the permeability to air is relatively high. As the sample size is increased, the length of drainage path also increases. In large samples it is possible that the induced air pressure may be an influential factor in the time-dependent behavior of compacted cohesive soils. So it was decided to investigate the effect of drainage on stress-strain-time response of the specimen.

Two series of tests were conducted to determine the effects of partial drainage and no drainage and the responses were compared with the regular creep tests which were fully drained. The zero drainage case corresponds to the case of an infinite drainage length. In the first series, samples were consolidated to the required pressure and then the drainage valves were closed when the creep loads were applied. This can be considered equivalent to partial drainage. In the second series unconfined samples were tested. Instead of porous stone,plexiglas disc was placed at the bottom of the specimen and the drainage valves were kept closed during the test. This is equivalent to fully undrained case. In both series filter drains were not used and it was assumed that there is no diffusion of air through the membrane.

As illustrated in Figure 4.8.1, the relation

$$(\varepsilon_1 - \varepsilon_3)(t) = ct^n$$

is valid for any drainage case. Further, the response of an individual sample is similar whether the sample is tested drained, partly drained or undrained, as indicated by Figure 4.8.2. Figures 4.8.3 and 4.8.4 show the comparison between partly drained and undrained cases with the fully drained case. The variations of parameter c and Poisson's ratio with stress ratio was essentially the same in the test series, irrespective of drainage conditions. It is evident that the response of compacted cohesive soil under load is essentially the same irrespective of the length of the drainage path. Hence it is concluded that pore air pressure does not affect

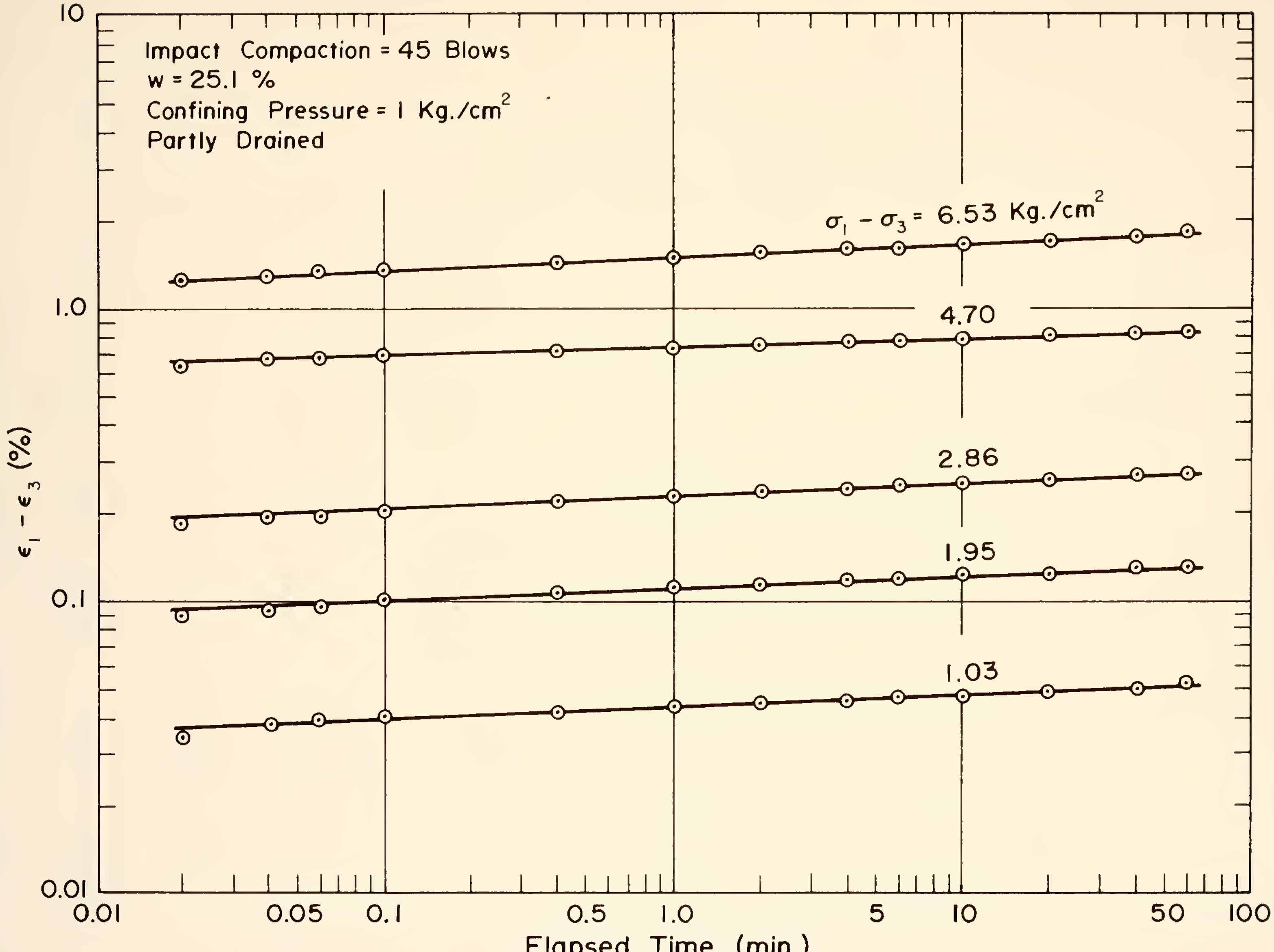
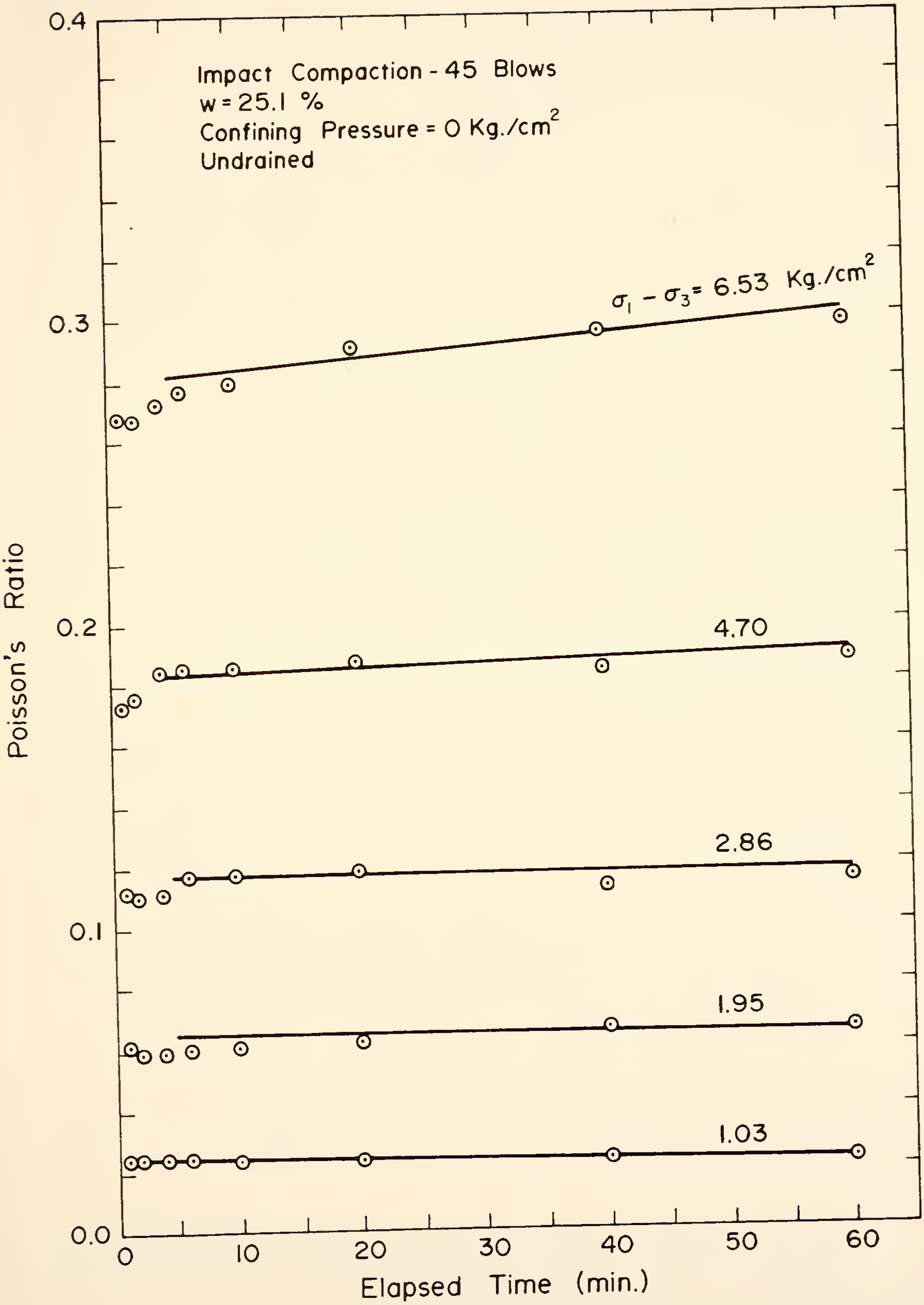


Figure 4.8.1 - Relation Between Shear Strain and Time for EPK



igure 4.8.2 - Relation Between Poisson s Ratio and Time for EPK

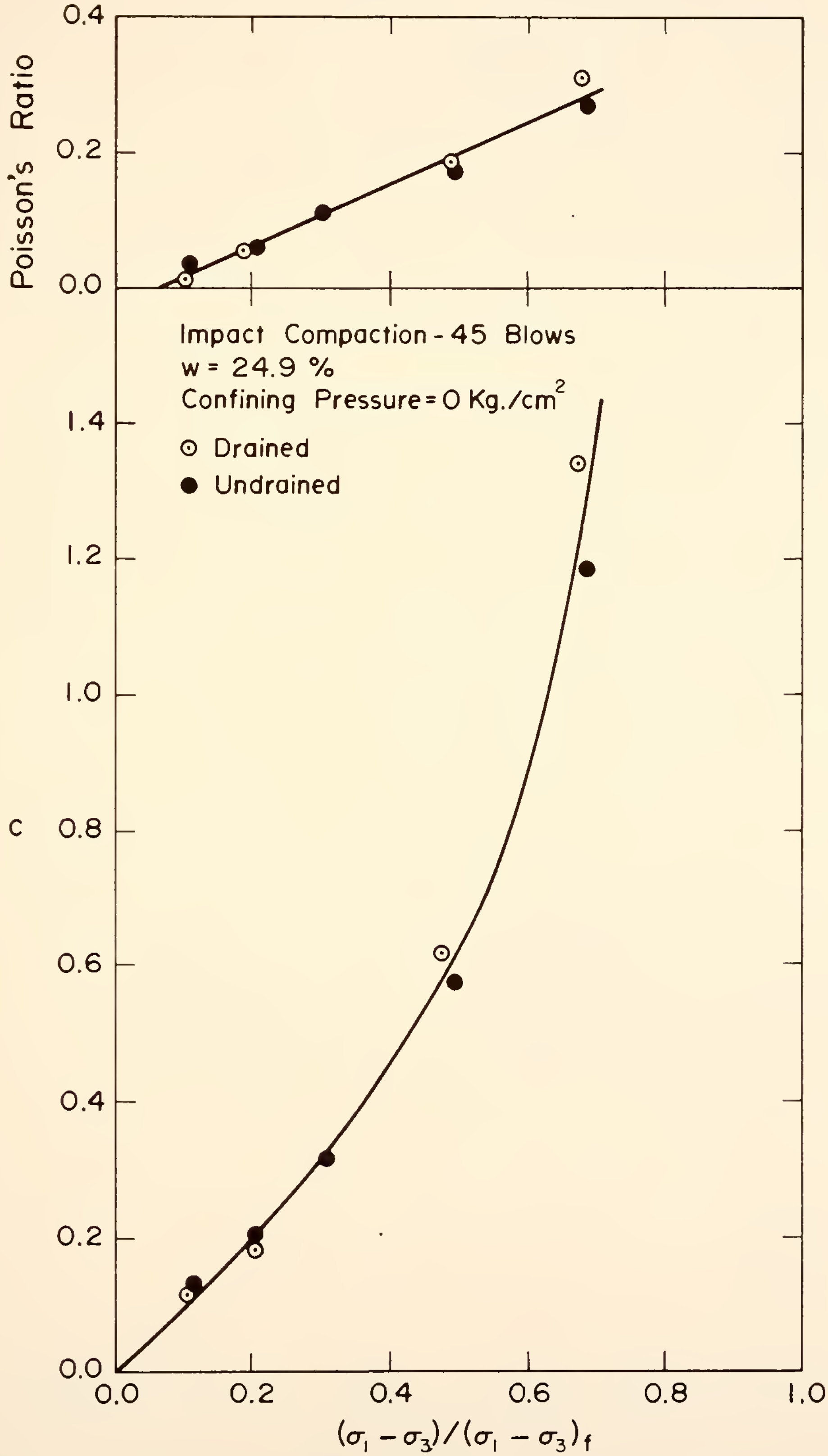
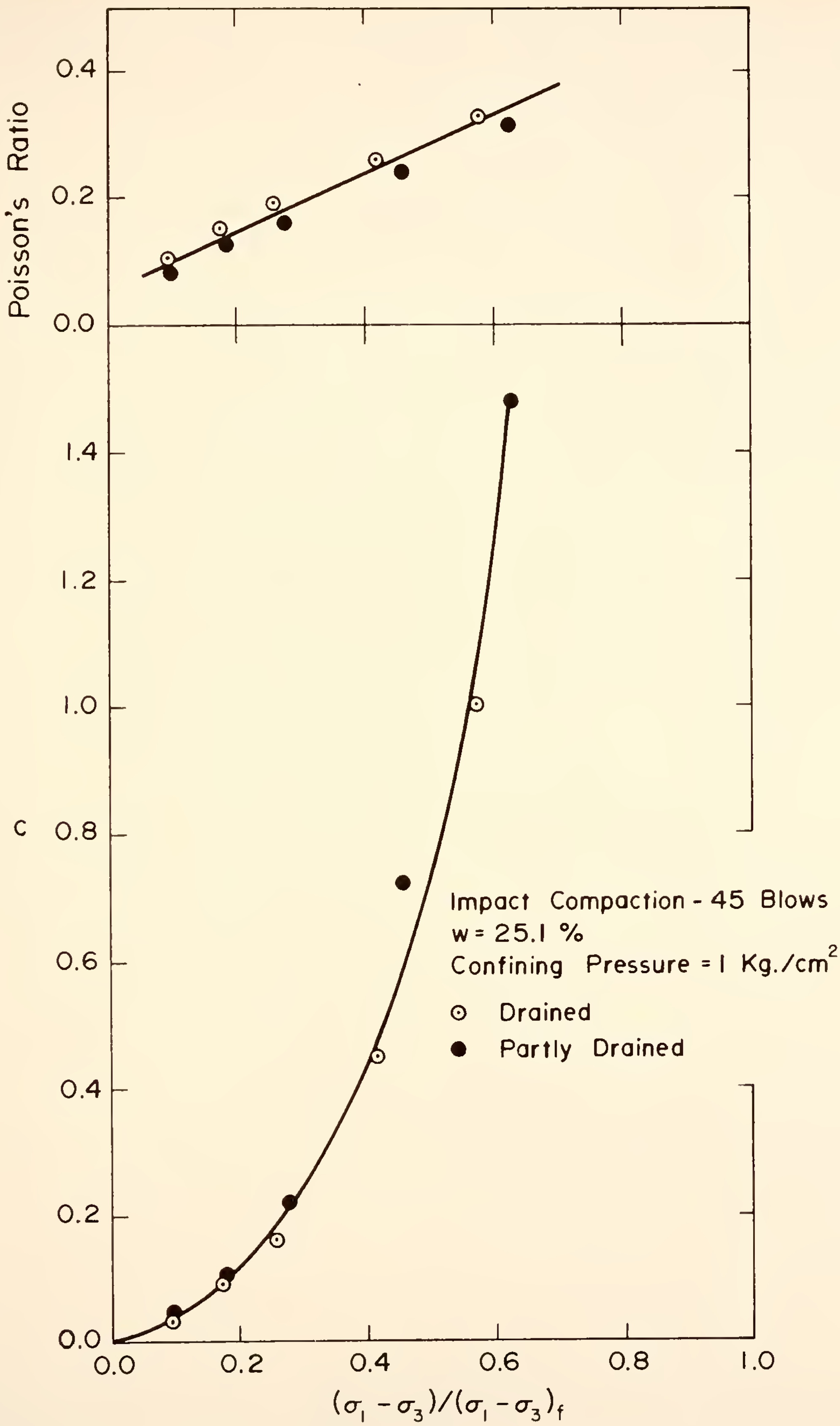


Figure 4.8.3 - Comparison of Effects of Stress Ratio for Drained and Undrained Creep Tests on EPK



igure 4.8.4 - Comparison of Effects of Stress Ratio for Drained and Partly Drained Creep Tests on EPK

significantly the time-dependent behavior of partly saturated cohesive soil in the range of stresses used, and the mechanical properties are independent of the size of the soil mass considered in testing.

4.9 - Generalized Stress-Strain-Time Relationship.

Attempts to establish a comprehensive stress-strain-time relationship for cohesive soils have all been generalized from relatively simple testing conditions. The most common testing procedure used is the triaxial compression test, in which radial symmetry exists. Even for this type of test there is no justification for assuming that the properties of a soil determined for a particular loading history will be valid for any other condition. Nonetheless it is useful to compare the test results obtained in this study with those of two other investigations.

Lara-Tomas (1961) applied a constant torsional shear stress on a hollow cylindrical compacted soil sample and measured the shear deformation with time. The applied shear stress was the maximum shear stress as well as the octahedral shear stress. He plotted log strain rate against log time and concluded that,

$$\frac{d\gamma}{dt} = at^{-m} \qquad (4.9.1)$$

in which

γ = shear strain = $\varepsilon_1 - \varepsilon_3$

a, m = constants

t = time in minutes

On integration of Equation 4.9.1,

$$\gamma(t) = \gamma_o + \frac{at^{-m+1}}{-m+1} \qquad (4.9.2)$$

Setting

$n = -m+1$

$c = \frac{a}{-m+1}$

$\gamma_o = 0$.

Equation 4.9.2 becomes

$$\varepsilon_1 - \varepsilon_3 = ct^n \qquad (4.9.3)$$

which is Equation 4.2.1. This suggests that Lara-Tomas' (1962) results are

consistent with those presented herein. A direct comparison with his findings is shown in Figure 4.9.1, in which the logarithm of strain rate is shown as a linear function of logarithm of time for creep tests on EPK. As the shear stress is increased, the straight line has a tendency to move up, the value of m being almost the same, independent of stress level. These results are similar to those obtained by Lara-Tomas (1962).

. Similar results were obtained by Mitchell et al (1969) for measured axial strains of saturated clay specimens subjected to triaxial compression creep testing (Equation 2.1.1). Because radial strains are proportional to axial strains for saturated specimens, their results also support those presented herein. The results obtained by Pagen and Jagannath (1967,1968) are also consistent with the representation proposed.

4.10 - Extension Tests.

It is known that undrained shear strength of saturated clays depends on the direction of the applied total principal stress changes during shear stress-strain characteristics of saturated clays are different in triaxial compression and extension tests (Skempton and Sowa, 1963). The direction of major principal stress during loading with respect to the direction of principal stresses during consolidation is also an extremely important factor in determining the stress-strain moduli of saturated clays.

In the triaxial compression creep tests on compacted cohesive soils the direction of major principal stress is usually the direction in which compaction stresses were applied. Because the compacted soil sample is overconsolidated when taken out of the mold, the initial stresses in the sample are anisotropic. Thus it seems likely that a change in the direction of principal stresses will have a definite influence in the stress-strain-time characteristics of such materials.

To determine the influence of the change in the direction of major principal stress, two series of extension tests were conducted in which the direction of major principal stress was parallel to the plane of compaction. The soil was EPK and both impact and kneading compaction were used.

For these tests the Norwegian triaxial shear equipment was modified to make the loading piston the same diameter as the sample so that an increase in fluid pressure around the sample would not affect the axial stress.

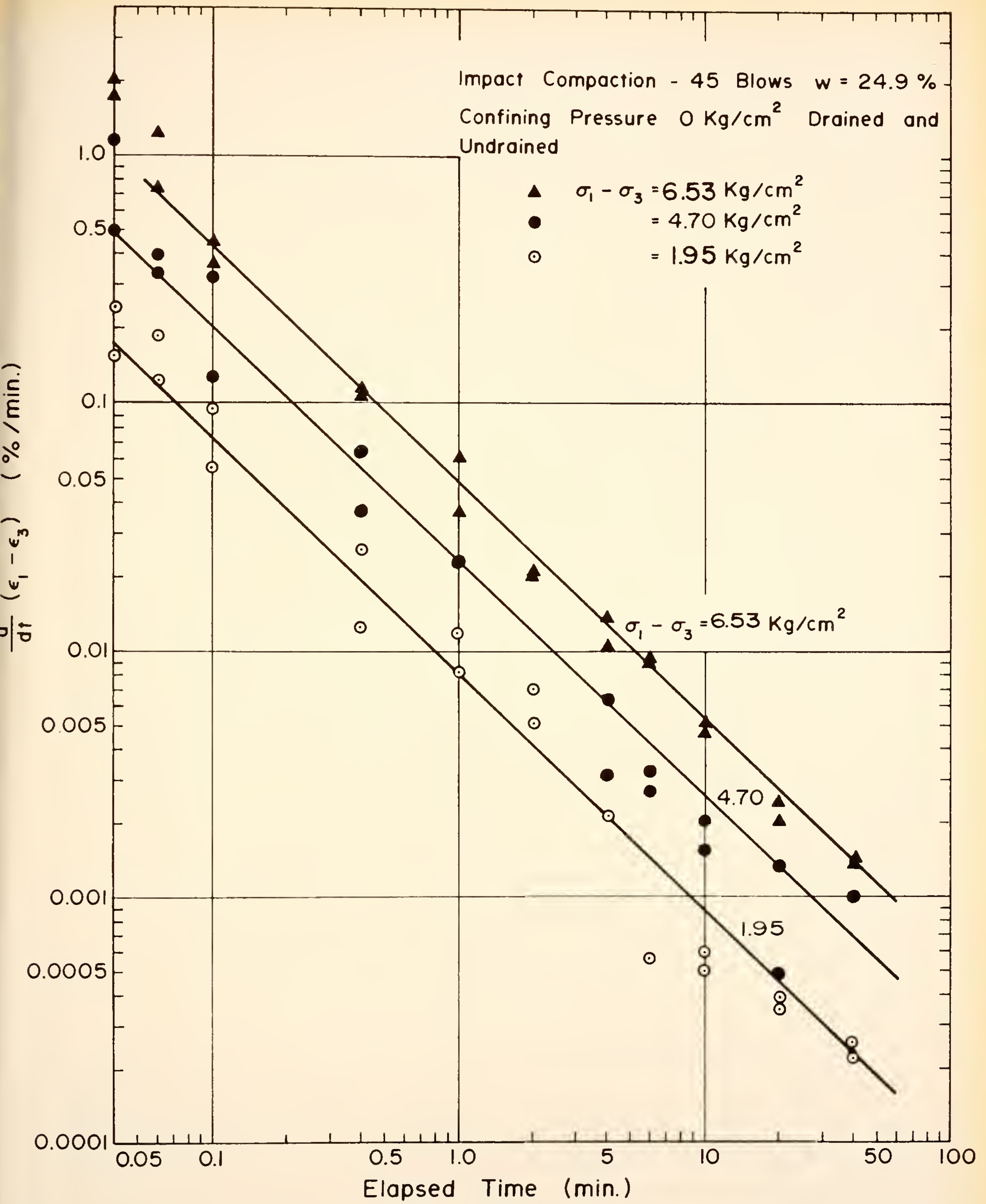


Figure 4.9.1 - Relation Between Shear Strain Rate and Time at Various Stress Levels for EPK

The triaxial extension tests were conducted undrained. The sample was set up in a generally similar manner as in compression creep tests. Confining pressure was applied by means of the constant pressure cell. The sample was thus isotropically consolidated to the required stress. The sample was allowed to consolidate for approximately twenty-four hours and then the lateral stress, which was the major principal stress, was increased to the required level.

When the lateral stress was applied, the triaxial cell itself deformed resulting in the movement top cap of the cell, which was the reference for measurement of axial deformation. Hence a correction had to be applied. This was done by conducting an extension test on a steel cylinder of the same dimensions as the sample. The stresses that were used were identical to the stress levels applied in the extension test; also they were too small to produce any deformation in the steel sample. Hence the movement of the axial deformation gauge was entirely due to the triaxial cell. Two tests were conducted and the average of the two was used to correct the extension test data on the compacted soil sample.

Since the major principal stresses in the extension test are the lateral stresses, the magnitude of the equivalent Poisson's ratio is not equal to the ratio between the principal strains. Its value at a particular time t can be obtained by noting that for isotropic linear elastic behavior:

$$\Delta\varepsilon_1 = \frac{1}{E}\left[\Delta\sigma_1 - \mu(\Delta\sigma_2 + \Delta\sigma_3)\right] \qquad (4.10.1)$$

and

$$\Delta\varepsilon_3 = \frac{1}{E}\left[\Delta\sigma_3 - \mu(\Delta\sigma_1 + \Delta\sigma_2)\right] \qquad (4.10.2)$$

can be simplified for the extension test for which

$$\Delta\sigma_2 = \Delta\sigma_1 = \Delta\sigma_r \qquad (4.10.3)$$

and

$$\Delta\sigma_3 = \Delta\sigma_a = 0 \qquad (4.10.4)$$

In these expressions the Δ indicates an incremental value after consolidation under an all-round pressure. Substituting Equations 4.10.3 and 4.10.4 into Equations 4.10.1 and 4.10.2, and dividing,

$$\frac{\Delta\varepsilon_3}{\Delta\varepsilon_1} = \frac{-2\mu}{1-\mu} \qquad (4.10.5)$$

or

$$\mu = \frac{-\Delta\varepsilon_3}{2\Delta\varepsilon_1 - \Delta\varepsilon_3} \qquad (4.10.6)$$

This expression was used to determine the equivalent Poisson's ratio at time t from the extension test results.

Figure 4.10.1 and 4.10.2 show the typical results of an extension test series conducted on compacted Edgar Plastic Kaolin (EPK). Figure 4.10.1 indicates that the relation between maximum shear strain and time in an extension test can be represented by Equation 4.2.1. The correlation coefficient for the results obtained exceeded 0.970 in all but one case.

As shown in Figure 4.10.2 the magnitude of Poisson's ratio was independent of time at a particular stress level. The influence of stress level on the equivalent Poisson's ratio was minor. However, the magnitude of the Poisson's ratio was greater than previously noted in the compression creep tests, even exceeding 0.5.

Figure 4.10.3, 4.10.4 show the comparison between drained compression creep tests and undrained extension tests for two types of compaction. At low shear stress levels the magnitudes of c are similar for the two tests. But at higher stress levels, the magnitude c for the extension tests was higher.

Due to limitations on the maximum chamber pressure, it was not possible to increase the stresses to failure in extension tests. Hence the results could not be compared at equal stress ratios. From an inspection of the figures, it seems probable that even if the stress ratio has been used, c would have been larger for the extension tests at higher stress ratios.

As shown in Figures 4.10.3 and 4.10.4, the magnitude of n was independent of the stress system and shear stress level. These results suggest that the soil behaves in an anisotropic manner. The anisotropy is induced by the compaction process. This view is supported by the fact that the difference between the parameters for extension and compression is somewhat greater for impact than kneading compaction. Impact compaction produces more unidirectional deformation than kneading.

The implication of anisotropic response is that a minimum of five parameters or functions are required to describe the mechanical behavior of the simplest anisotropic elastic material. Because of the extensive kneading

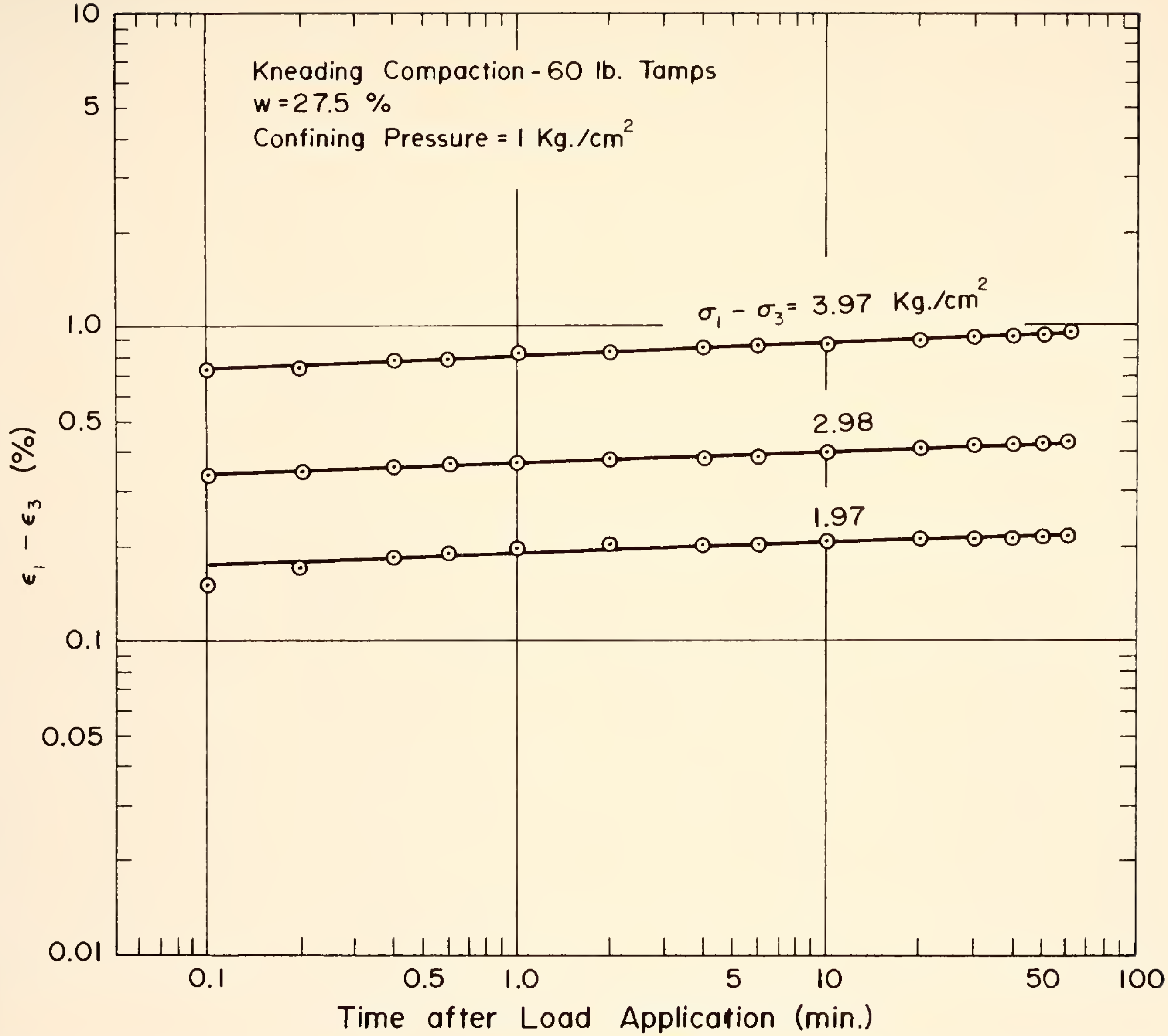


Figure 4.10.1 - Relation Between Maximum Shear Strain and Time for Extension Creep Tests on E P K

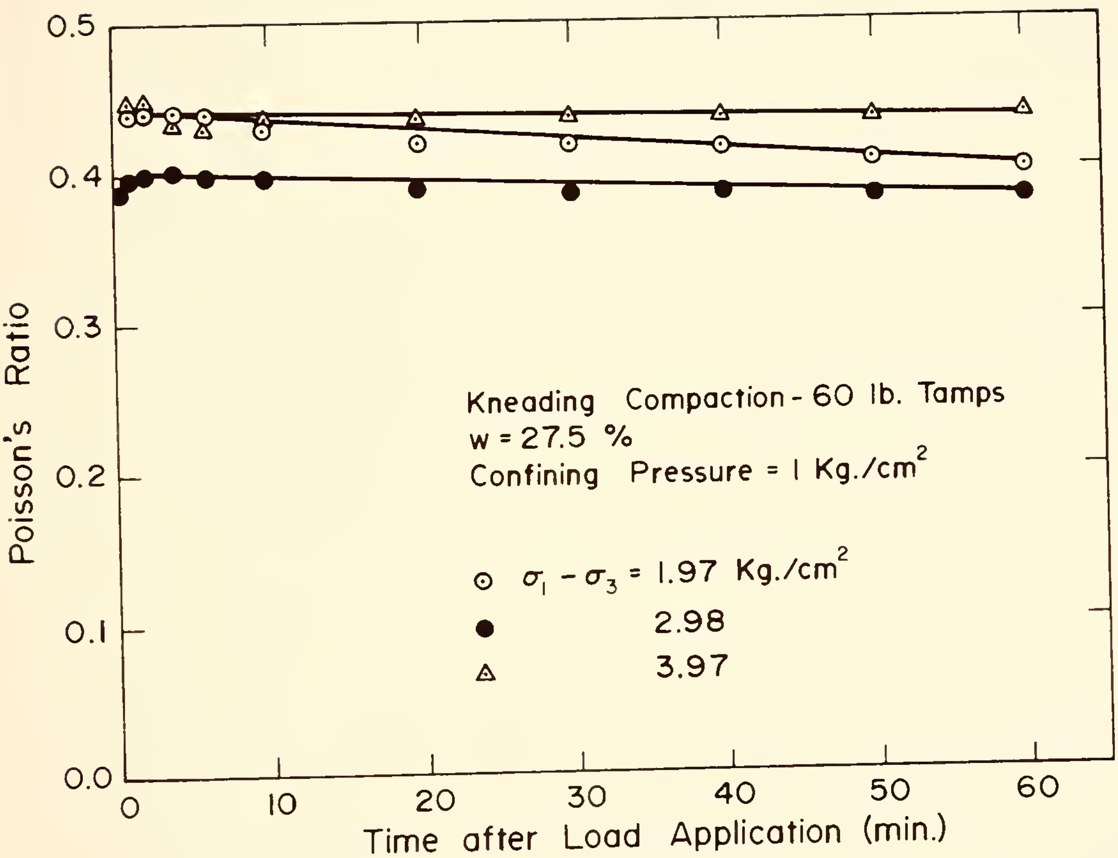


Figure 4.10.2 - Relation Between Poisson s Ratio and Time for Extension Tests on E PK

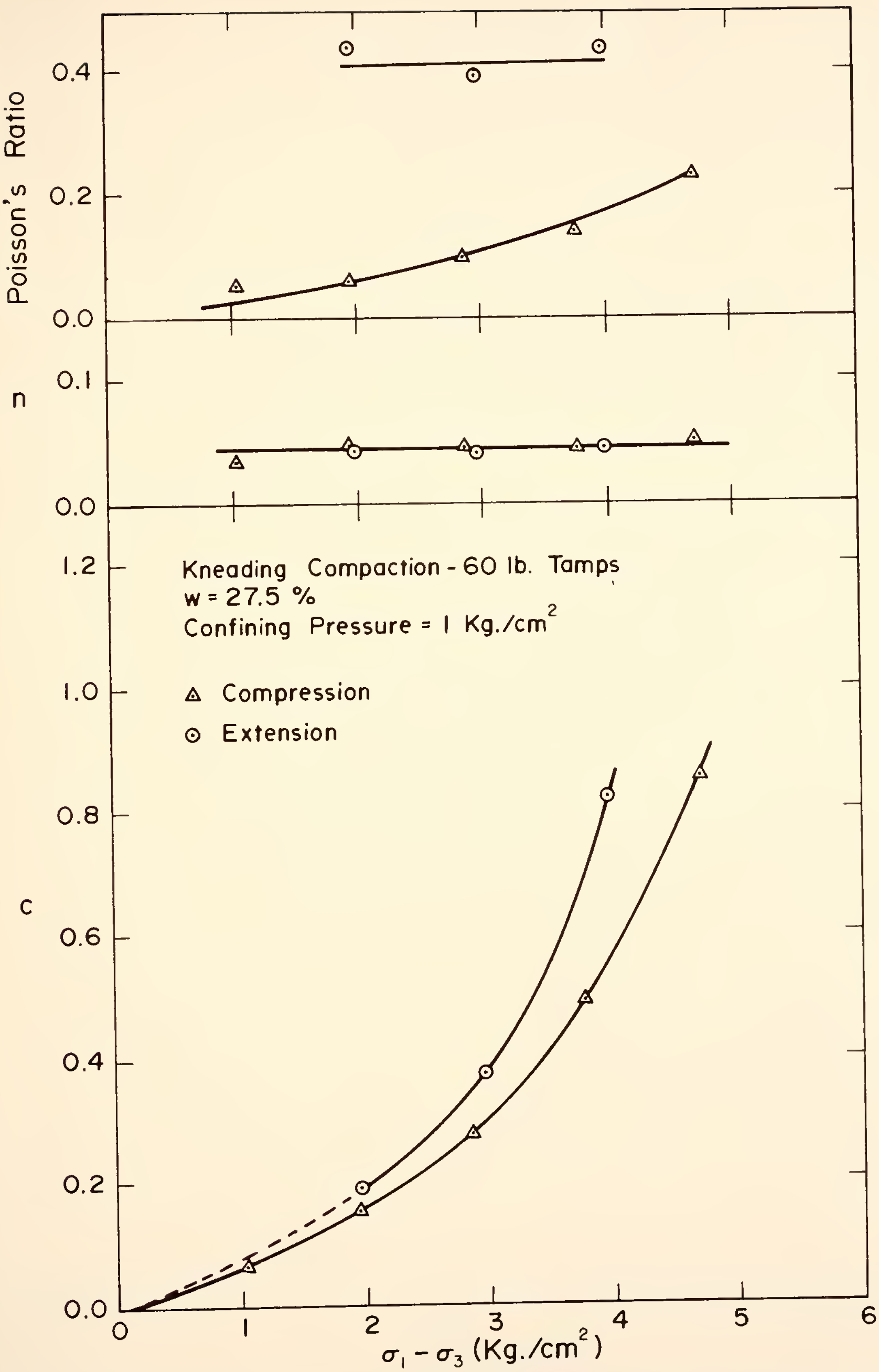
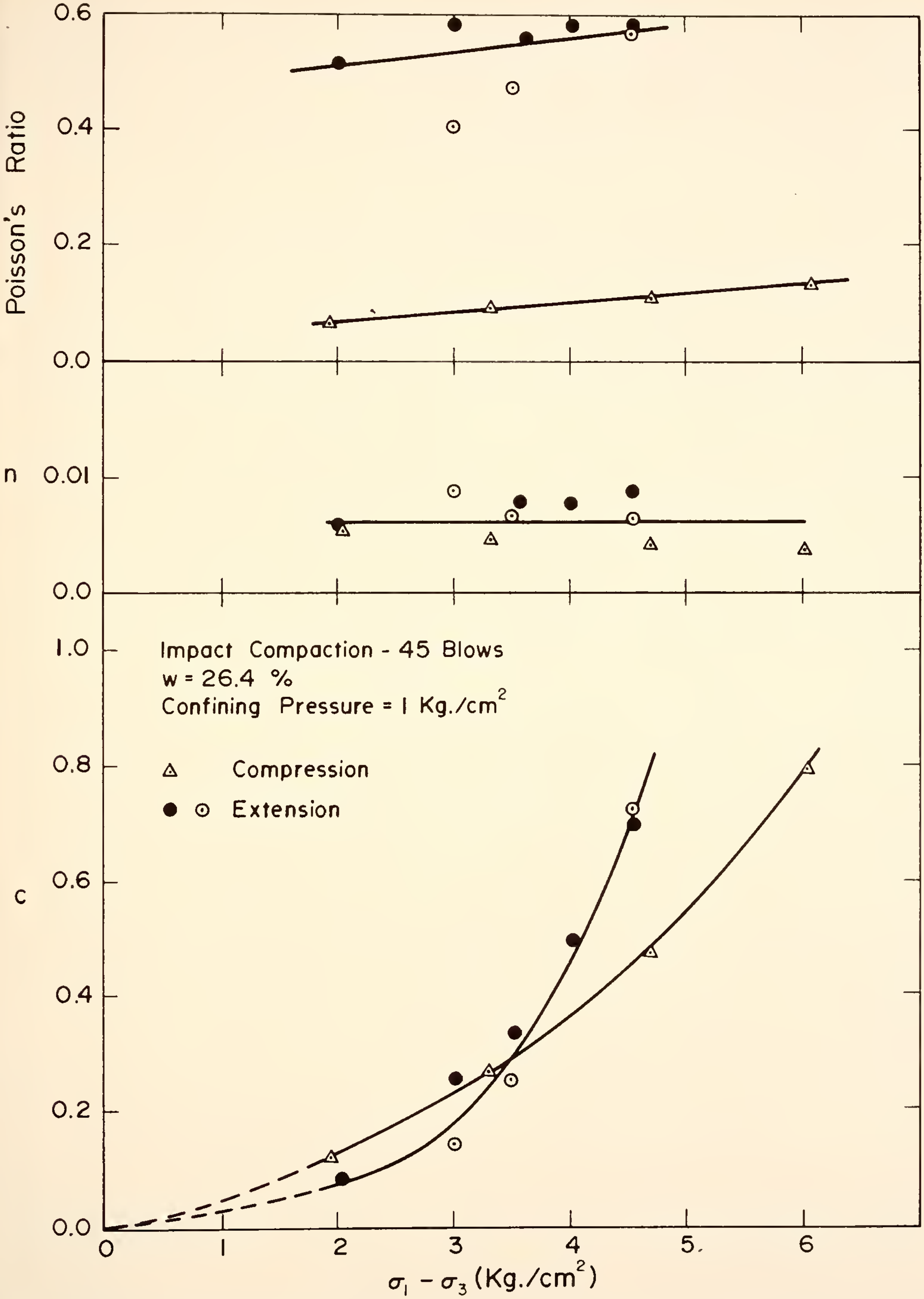


Figure 4.10.3 - Comparison of Effect of Stress Level on Creep Parameters Between Compression and Extension Creep Tests



gure 4.10.4 - Comparison of Effect of Stress Level on Creep Parameters Between Compression and Extension Tests for EPK

action of sneepsfoot roller during field compaction, it is not clear what degree of anisotropy would be evident in the field compacted material. For this reason, the results of compression tests will be assumed valid for isotropic material response in subsequent discussion.

4.11 - Nonlinear Viscoelastic Material Characterization.

Having identified the parameters which influence the time dependent behavior of compacted clays, it is then necessary to express this response in the form of a constitutive law which can be incorporated in an analysis of embankment deformations. This is done by generalizing from a linear to a nonlinear viscoelastic formulation.

By using the principles of superposition, linear viscoelastic behavior of any material, say for uniaxial testing, can be expressed by the Boltzman hereditary integrals:

$$\varepsilon(t) = \int_{-\infty}^{t} D(t-\tau)\, \frac{d\sigma(\tau)}{d\tau}\, d\tau \qquad (4.11.1)$$

or

$$\sigma(t) = \int_{-\infty}^{t} E(t-\tau)\, \frac{d\varepsilon(\tau)}{d\tau}\, d\tau \qquad (4.11.2)$$

where

$D(t-\tau)$ = uniaxial creep compliance function

$E(t-\tau)$ = uniaxial relaxation modulus function

$\varepsilon(t)$ = uniaxial strain

$\sigma(t)$ = uniaxial stress

t = time

These integral equations are not independent and one can be derived from the inversion of the other. They can be related by,

$$E(0)\, D(t) = 1 + \int_{0}^{t} D(\tau)\, \frac{dE(t-\tau)}{d(t-\tau)}\, d\tau \qquad (4.11.3)$$

where

$E(0)$ = initial value of relaxation modulus;

$D(t)$ = creep compliance.

It is obvious that the above equation holds even if D and E are interchanged.

Considering a uniaxial compression creep test with constant stress σ_o applied at t = 0 to an initially unstressed and unstrained body, i.e.,

$$\sigma(0^-) = \varepsilon(0^-) = 0$$

then

$$\varepsilon(t) = \sigma_o D(0) + \sigma_o \int_0^t \frac{dD(t-\tau)}{d(t-\tau)} d\tau \qquad (4.11.4)$$

Many viscoelastic materials respond in a linear manner at the stress levels encountered in engineering applications. However, some polymers and concrete exhibit nonlinear behavior at high stresses relative to failure conditions. As shown above, compacted clays exhibit marked nonlinearity even at low stresses and strains.

Based on the principles of irreversible thermodynamics, Schapery (1969a, 1969b), and Lou and Schapery (1969) have shown that the response of many types of nonlinear viscoelastic materials can be expressed in a form similar to that for linear viscoelastic behavior (Equations 4.11.1 or 4.11.2). For example, for uniaxial loading,

$$\varepsilon = g_o D_o \sigma + g_1 \int_0^t \Delta D(\psi - \psi') \frac{d\, g_2\sigma}{d\tau} d\tau \qquad (4.11.5)$$

where

D_o and $\Delta D(\psi)$ are the components of linear viscoelastic creep compliance;

ψ is the "reduced" time defined by

$$\psi = \int_0^t \frac{dt'}{a_\sigma} \qquad a_\sigma > 0 \qquad (4.11.6)$$

$$\psi' = \psi(\tau) = \int_0^\tau \frac{dt'}{a_\sigma} \qquad (4.11.7)$$

g_o, g_1, g_2 and a_σ are material properties which are functions of stress.

These stress dependent properties have specific thermodynamic significance: changes in g_o, g_1 and g_2 reflect third and higher order dependence of the Gibb's energy on the applied stress and a_σ arises from similar higher order effects in both entropy production and free energy. It is possible to determine whether the nonlinearity exists in elastic or viscous components by studying the behavior of these parameters.

Using creep and recovery data the magnitudes of g_o, g_1, g_2 and a_σ can be determined. For a two step uniaxial loading test,

$$\sigma = \begin{cases} 0 & t < 0 \\ \sigma_a & 0 < t < t_a \\ \sigma_b & t_a < t < t_b \end{cases} \qquad (4.11.8)$$

where σ_a and σ_b are constants. Then from Equation 4.11.5 for $0 < t < t_a$, using the Dirac delta function for $d\sigma/dt$:

$$\varepsilon = [g_o^a\, D_o + g_1^a\, g_2^a\, \Delta D(\psi)]\, \sigma_a \qquad (4.11.9)$$

or

$$\varepsilon = D_n^a\, \sigma_a \qquad (4.11.10)$$

in which

$$\psi = \frac{t}{a_\sigma^a}$$

and D_n^a is the nonlinear uniaxial creep compliance for the stress σ_a. The strain for $t_a < t < t_b$ is

$$\varepsilon = g_o^b\, D_o\, \sigma_t + g_1^b\, [g_2^a\, \sigma_a\, \Delta D(\psi_b)$$

$$+ (g_2^b\, \sigma_b - g_2^a\, \sigma_a)\, \Delta D(\psi_a)] \qquad (4.11.11)$$

in which

$$\psi_a = \frac{t - t_a}{a_\sigma^b}$$

$$\psi_b = \frac{t_a}{a_\sigma^a} + \frac{t - t_a}{a_\sigma^b}$$

The superscripts on the material parameters indicate the stress at which they are to be evaluated.

These equations can be used to evaluate the parameters g_o, g_1, g_2, a_σ from creep and recovery data. For a creep and recovery test

$$\sigma_a = \sigma_1 - \sigma_3 \qquad (4.11.12)$$

$$\sigma_b = 0 \qquad (4.11.13)$$

Combining Equations 4.11.9, 4.11.10 and 4.11.12 and expressing them in terms of shear strains, the creep strain is

$$\varepsilon_1 - \varepsilon_3 = J_n\,(\sigma_1 - \sigma_3) = [g_o\,J_o + g_1\,g_2\,\Delta J(\psi)](\sigma_1 - \sigma_3) \quad (4.11.14)$$

in which: $\Delta J(\psi)$ is the linear shear creep compliance

J_o is the initial (time independent) component of the shear creep compliance

J_n is the nonlinear shear creep compliance at shear stress level $\sigma_1 - \sigma_3$

From this expression, the time dependent component of the shear creep compliance is

$$\Delta J_n = J_n - g_o\,J_o = g_1\,g_2\,\Delta J(\psi) \quad (4.11.15)$$

or

$$\log\,(\Delta J_n) = \log\,(g_1\,g_2) + \log\,\Delta J(\psi) \quad (4.11.16)$$

Thus in general, if log (ΔJ_n) is plotted as a function of log(t), curves at different stress levels should be parallel. Using the reference linear viscoelastic compliance curve as a "master curve", the others can be shifted to it. The amount of horizontal (t) shift is equal to log a_σ, and the vertical shift is log $(g_1\,g_2)$.

If, however, the lines at different stress levels are parallel and straight, as indicated by the test results on compacted cohesive soils, unique values for a_σ and $g_1\,g_2$ cannot be obtained from creep data alone. Recovery data must also be used.

Denoting the recovery shear strain by ε_r,

$$\varepsilon_r \equiv (\varepsilon_1 - \varepsilon_3)_{recovery}$$

the recovery curve, in terms of shear parameters, for unloading at $t = t_a$ is from Equation 4.11.11 ($g_1^b = 1$)

$$\varepsilon_r = g_2(\sigma_1 - \sigma_3)[\Delta J(\psi_b) - \Delta J(\psi_b - \psi_a)] \quad (4.11.17)$$

in which

$$\psi_b = \frac{t_a}{a_\sigma} + t - t_a; \quad \psi_a = \frac{t_a}{a_\sigma}$$

For the compacted clays tested it is apparent from Equation 4.2.1 that

$$J_o = 0\ , \qquad \Delta J(\psi) = k\,\psi^n \quad (4.11.18)$$

in which k is a constant for the linear creep compliance. Then the creep strain at $t = t_a$, $\Delta\varepsilon_c$, is

$$\Delta\varepsilon_c = g_1\, g_2\, k \left(\frac{t_a}{a_\sigma}\right)^n (\sigma_1 - \sigma_3) \qquad (4.11.19)$$

and the recovery strain is

$$\varepsilon_r = g_2\, (\sigma_1 - \sigma_3)\, k[\psi_b^n - (\psi_b - \psi_a)^n] \qquad (4.11.20)$$

The ratio of Equations 4.11.20 and 4.11.19 is

$$\frac{\varepsilon_r}{\Delta\varepsilon_c} = \frac{1}{g_1}\,[(1 + a_\sigma\lambda)^n - (a_\sigma\lambda)^n] \qquad (4.11.21)$$

in which

$$\lambda = \frac{t - t_a}{t_a}$$

Rewriting Equation 4.11.21,

$$\mathrm{Log}\left(\frac{\varepsilon_r}{\Delta\varepsilon_c}\right) = -\log g_1 + \log [(1 + a_\sigma\lambda)^n - (a_\sigma\lambda)^n] \qquad (4.11.22)$$

It is evident from Equation 4.11.22 that a graphical shifting procedure can be applied to recovery data to determine a_σ and g_1, for different stress levels. A reference curve of normalized recovery strain $(\varepsilon_r/\Delta\varepsilon_c)$ is plotted against λ on double logarithmic paper by letting $a_\sigma = g_1 = 1$ in Equation 4.11.22 for appropriate values of n. The reference curve can be considered as the normalized recovery strain for a linear viscoelastic material. The normalized recovery curve for each stress level can be superimposed on the reference linear viscoelastic curve by translating them along the two axes. The amount of horizontal shift (λ)(to the left) and the vertical $(\varepsilon_r/\Delta\varepsilon_c)$ shift (up) are equal to log a_σ and log g_1 respectively. Since the curves are not generally straight lines unique values of a_σ and g_1 can be obtained. The magnitude of g_2 is found from the creep curves. Knowing the value of a_σ, g_1 and the linear viscoelastic creep compliance, different values of g_2 can be substituted in Equation 4.11.14 until the predicted creep curve coincides with the actual creep curve. Hence all the values that are needed to characterize the material at a particular stress level can be obtained.

As a first step in verifying the nonlinear viscoelastic constitutive equations are applicable to compacted cohesive soils, a series of creep test

results for Grundite, compacted by kneading compaction, was used. Master recovery curves are shown for two values of n in Figure 4.11.1. By superposing the actual recovery curves on top of the master recovery curves, the values of g_1 and a_σ for each stress level were obtained, as for example in Figure 4.11.2. It was found that even at the lowest stress level the value of g_1 was not equal to one. Hence linear viscoelastic stress level was lower than the lowest stress level used.

The maximum shear strain is given by Equation 4.11.14. Since the linear viscoelastic stress range was not known, the magnitude of k was not known. But it was possible to obtain the value of g_2k for each stress level. Knowing the values of g_1 and a_σ, trial values of g_2k were fed into the Equation 4.11.14 until the best coincidence was obtained between the computed plot of shear strain versus time and the actual creep curve obtained for that particular stress level.

A comparison of predicted and measured results is shown in Figure 4.11.3. The maximum error in prediction was about five per cent at the largest stress. The values of g_1, g_2k and a_σ used are plotted against $(\sigma_1 - \sigma_3)/(\sigma_1 - \sigma_3)_f$ as shown in Figure 4.11.4.

Two additional series of tests were conducted to investigate the applicability of the approach described above. Samples of EPK, both impact and kneading compacted were used. The procedure adopted was as follows.

The triaxial creep and recovery test was performed on an unconfined sample for each compaction type. By superposing the recovery curves on the reference curves the values of g_1, and a_σ were determined for each stress level as illustrated in Figure 4.11.5. Then with these values of a_σ and g_1 suitable value of g_2k was obtained by trial and error. The values of g_1, g_2k and a were plotted against $(\sigma_1 - \sigma_3)/(\sigma_1 - \sigma_3)_f$ as shown in Figures 4.11.6 and 4.11.7. To predict the response of a different specimen values of g_1, g_2k and a_σ were taken from these figures for the particular value of $(\sigma_1 - \sigma_3)/(\sigma_1 - \sigma_3)_f$. Then by using the equations

$$\varepsilon_1 - \varepsilon_3 = g_1 g_2 k \left(\frac{t}{a_\sigma}\right)^n \qquad (4.11.23)$$

and

$$\varepsilon_r = \frac{\Delta\varepsilon_c}{g_1}\left[(1 + a_\sigma\lambda)^n - (a_\sigma\lambda)^n\right] \qquad (4.11.24)$$

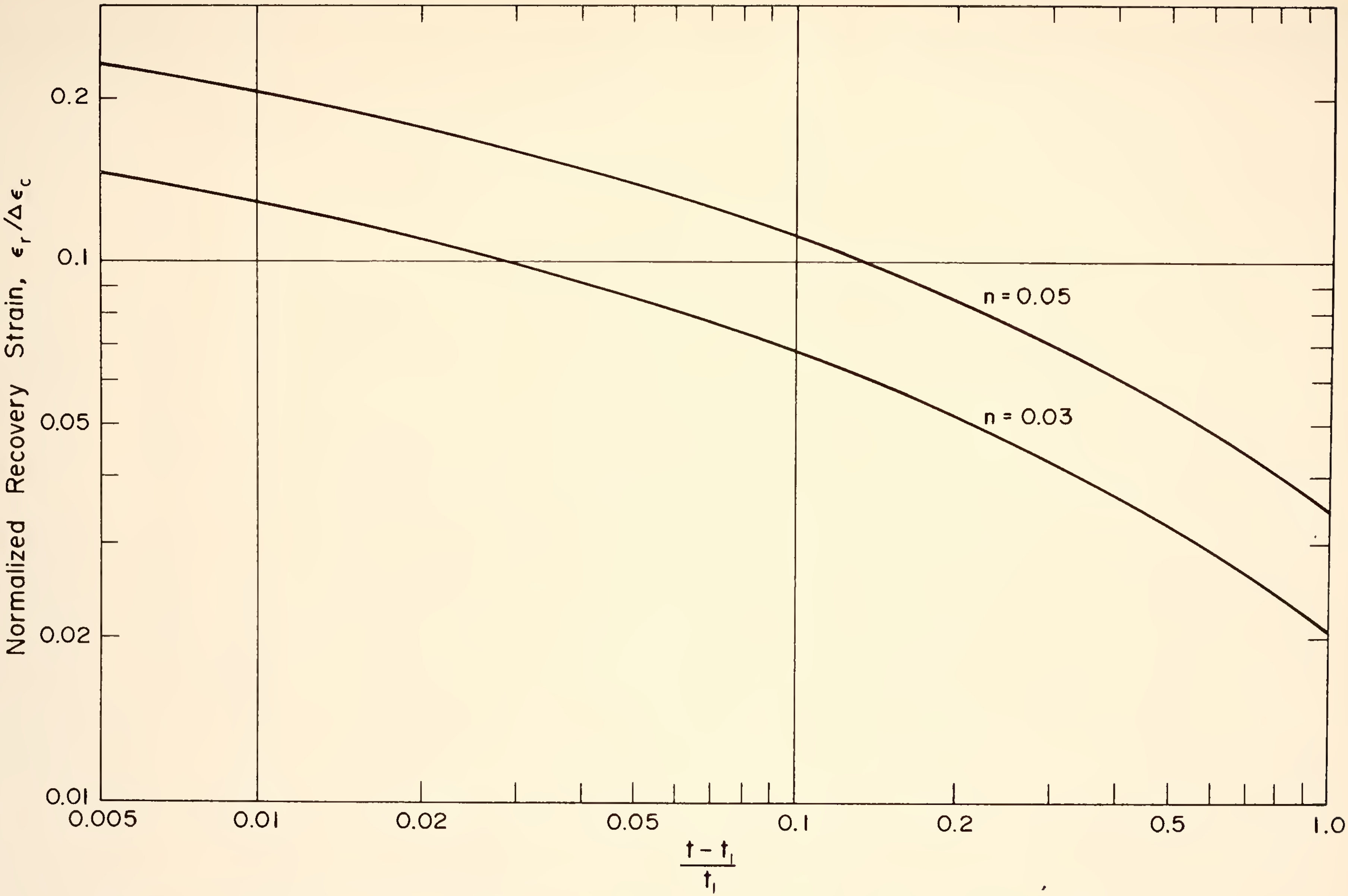


Figure 4.11.1 - Normalized Recovery Curve for $J(t) = J(0) + J_1 t^n$

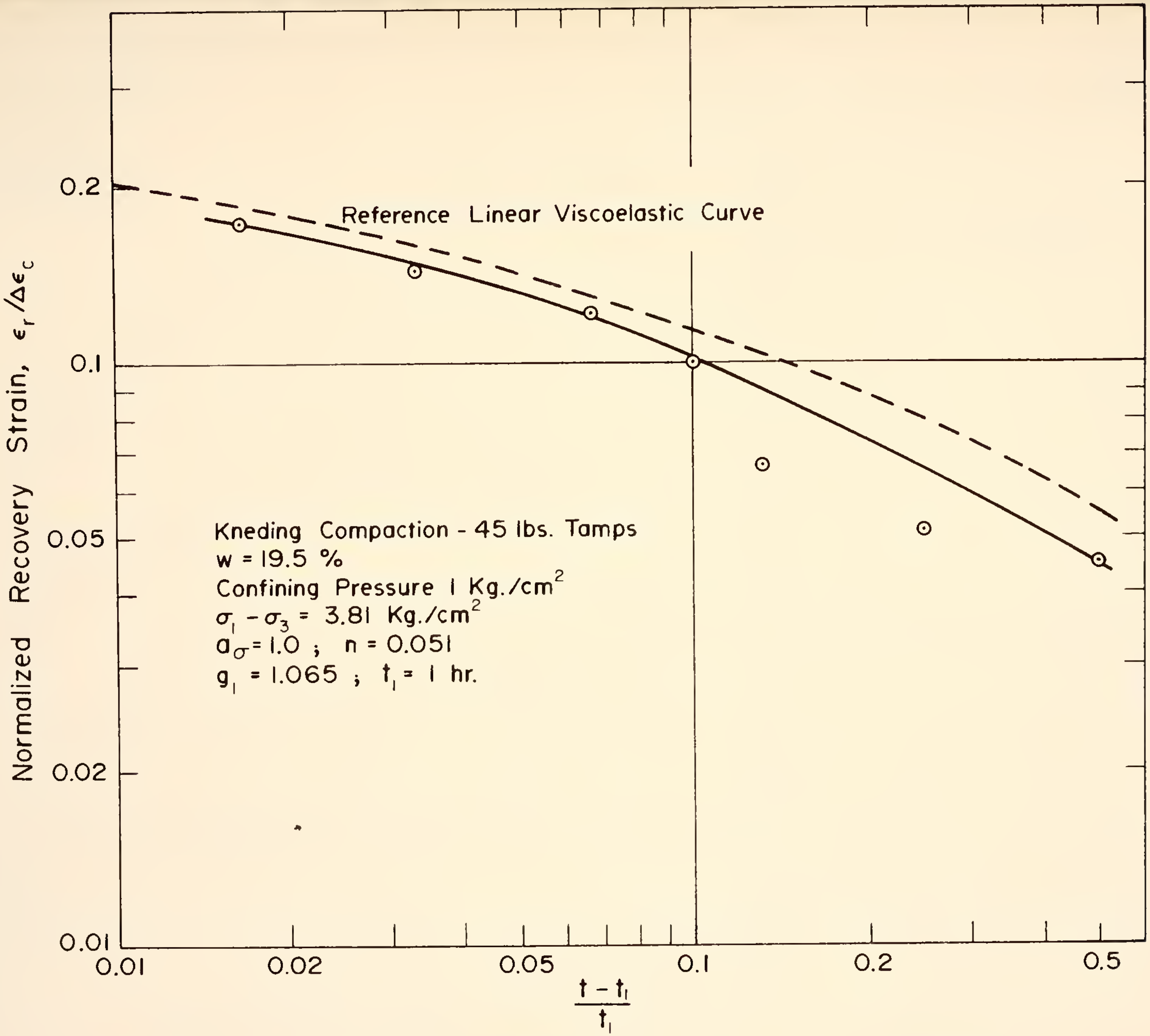


Figure 4.11.2 - Normalized Recovery Curve for Grundite

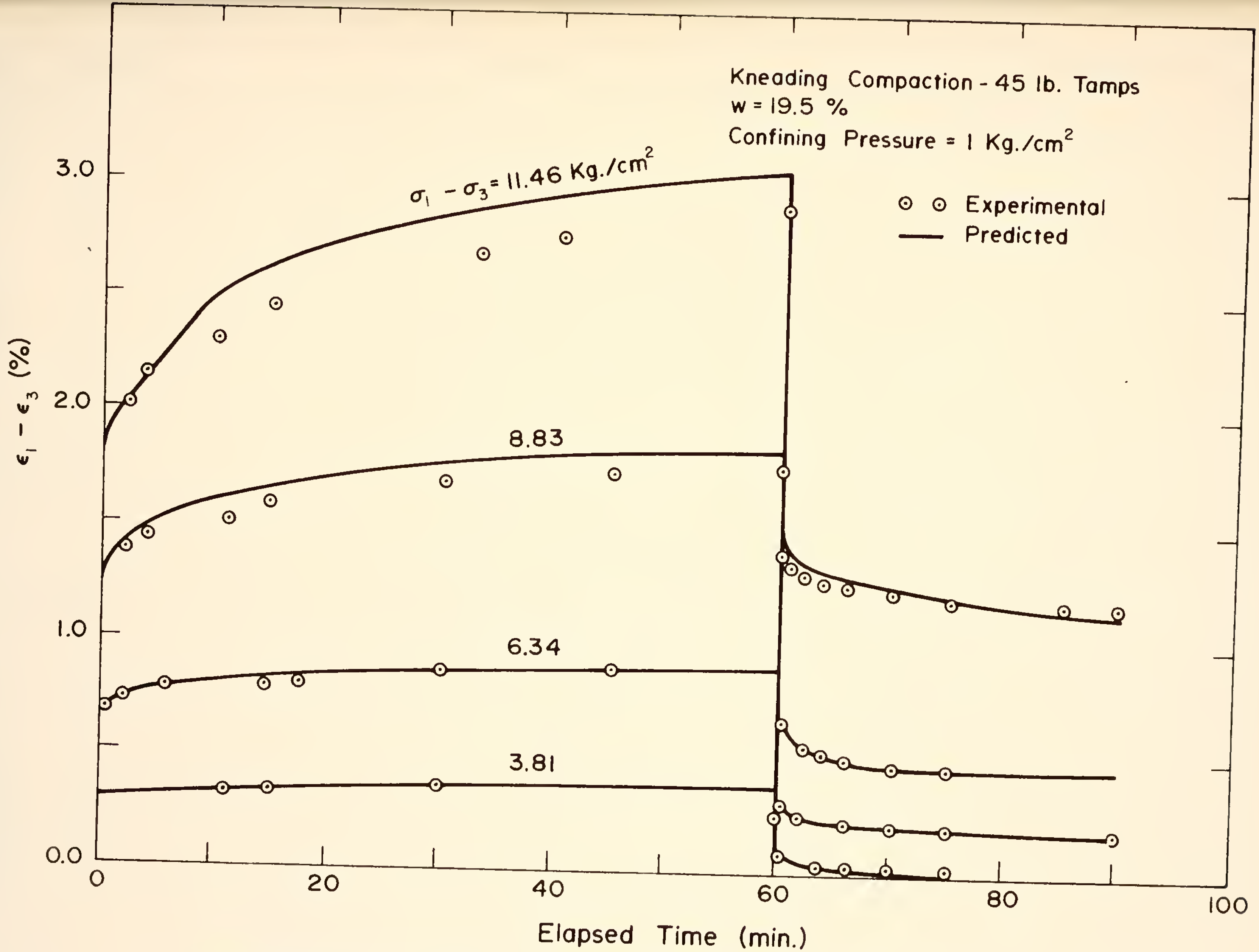


Figure 4.11.3 - Creep and Recovery for Grundite

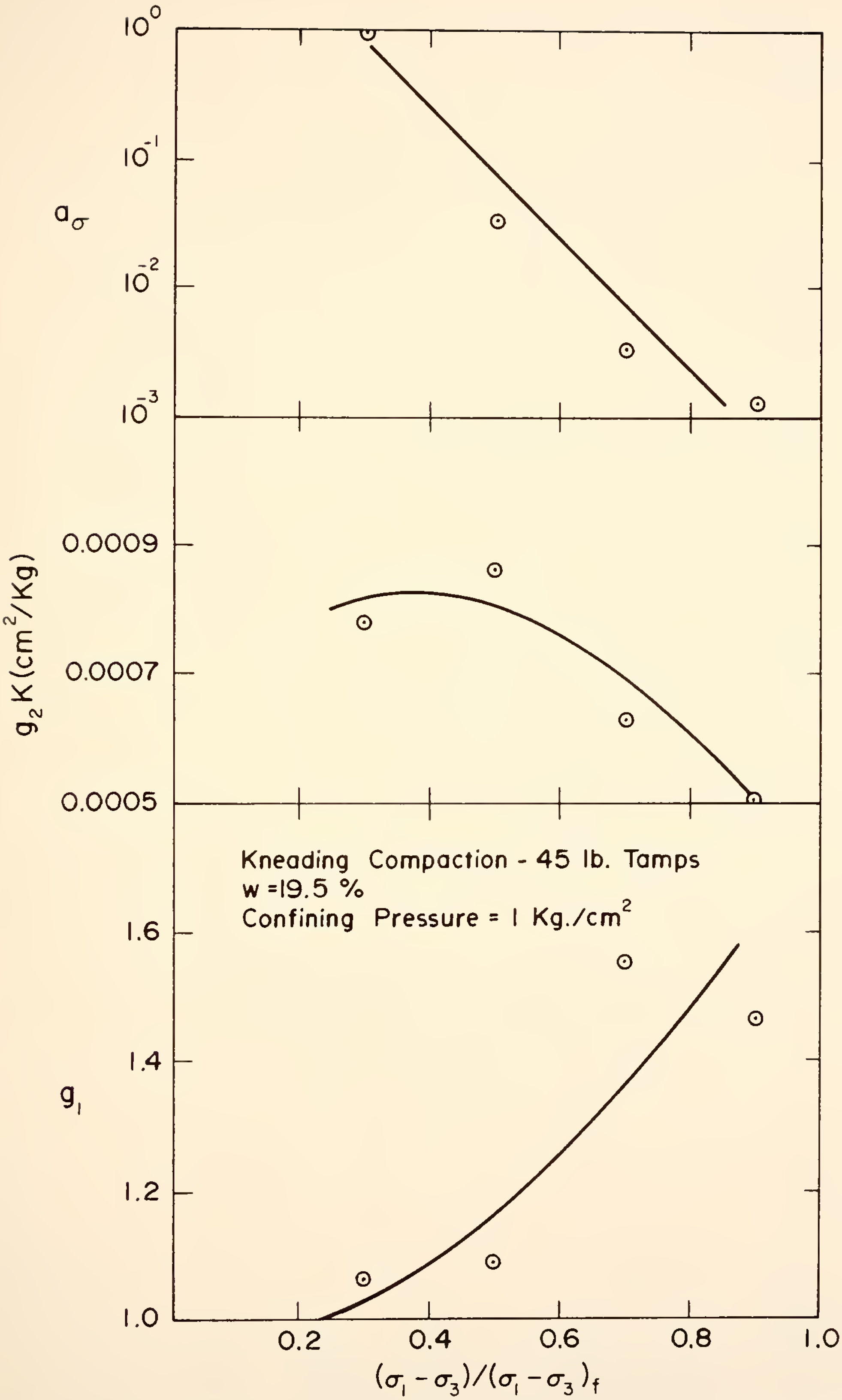


Figure 4.11.4 - Effect of Stress Ratio on Viscoelastic Parameters for Grundite

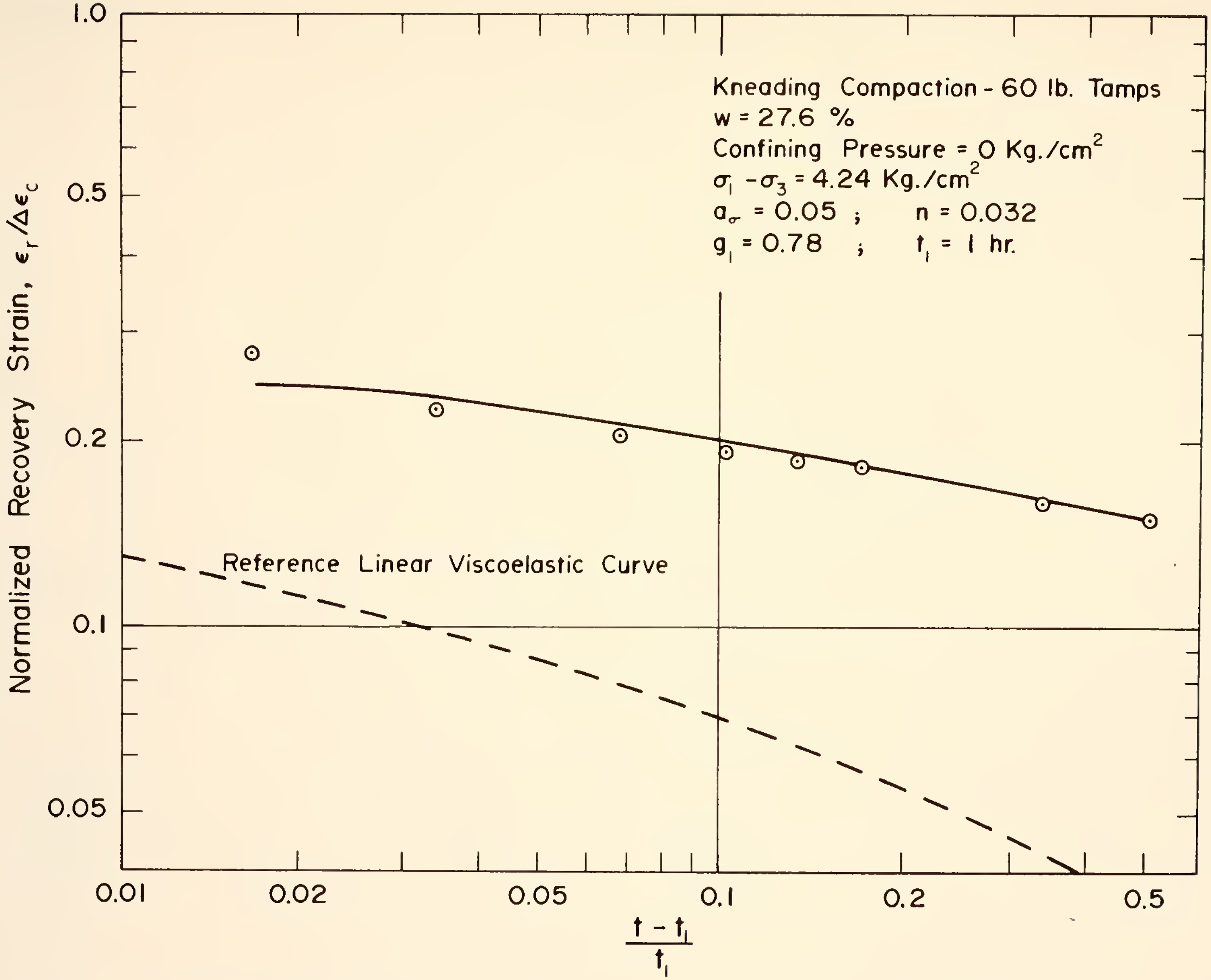


Figure 4.11.5 - Normalized Recovery Curve for EPK

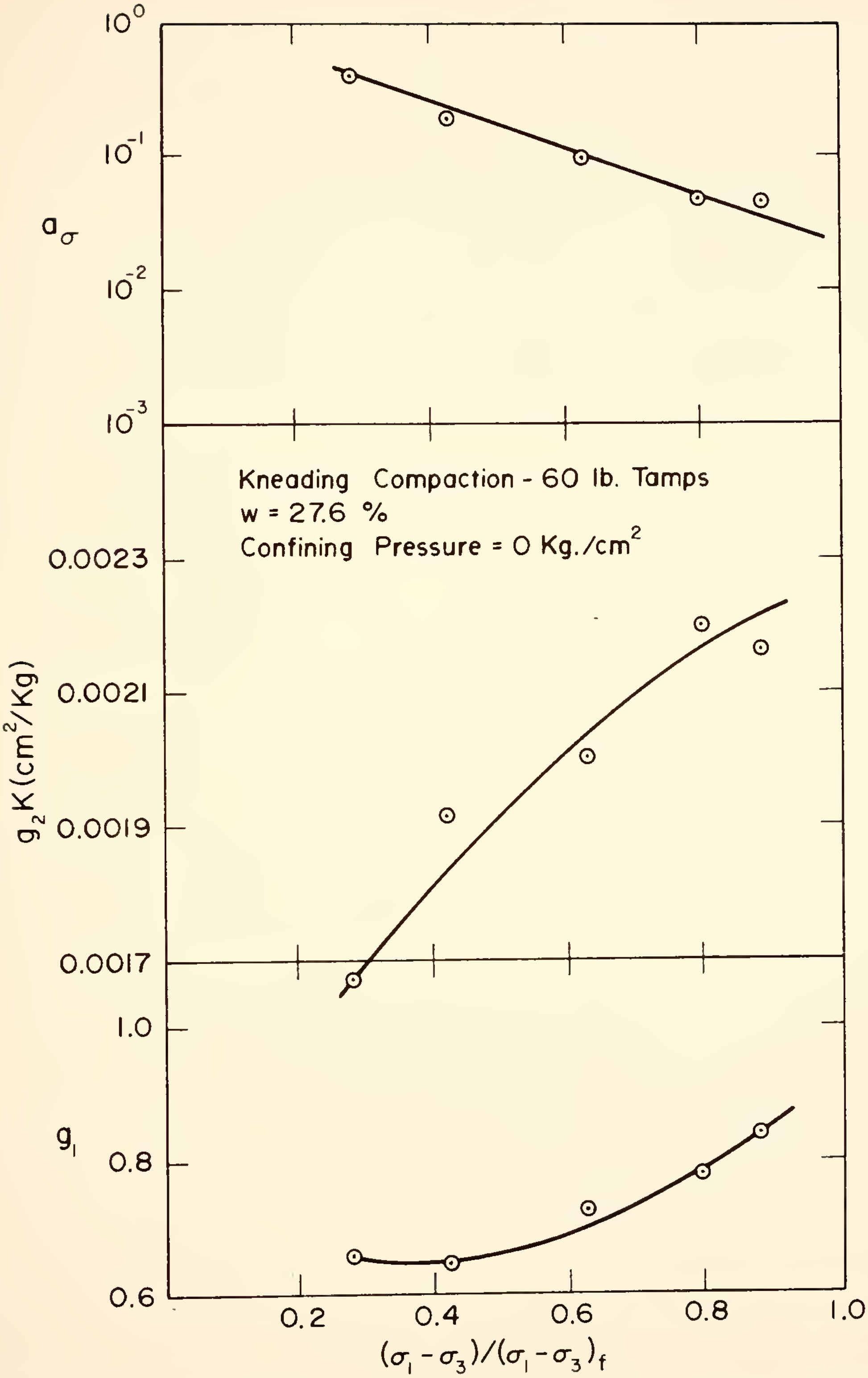


Figure 4.11.6 - Effect of Stress Ratio on Viscoelastic Parameters for EPK

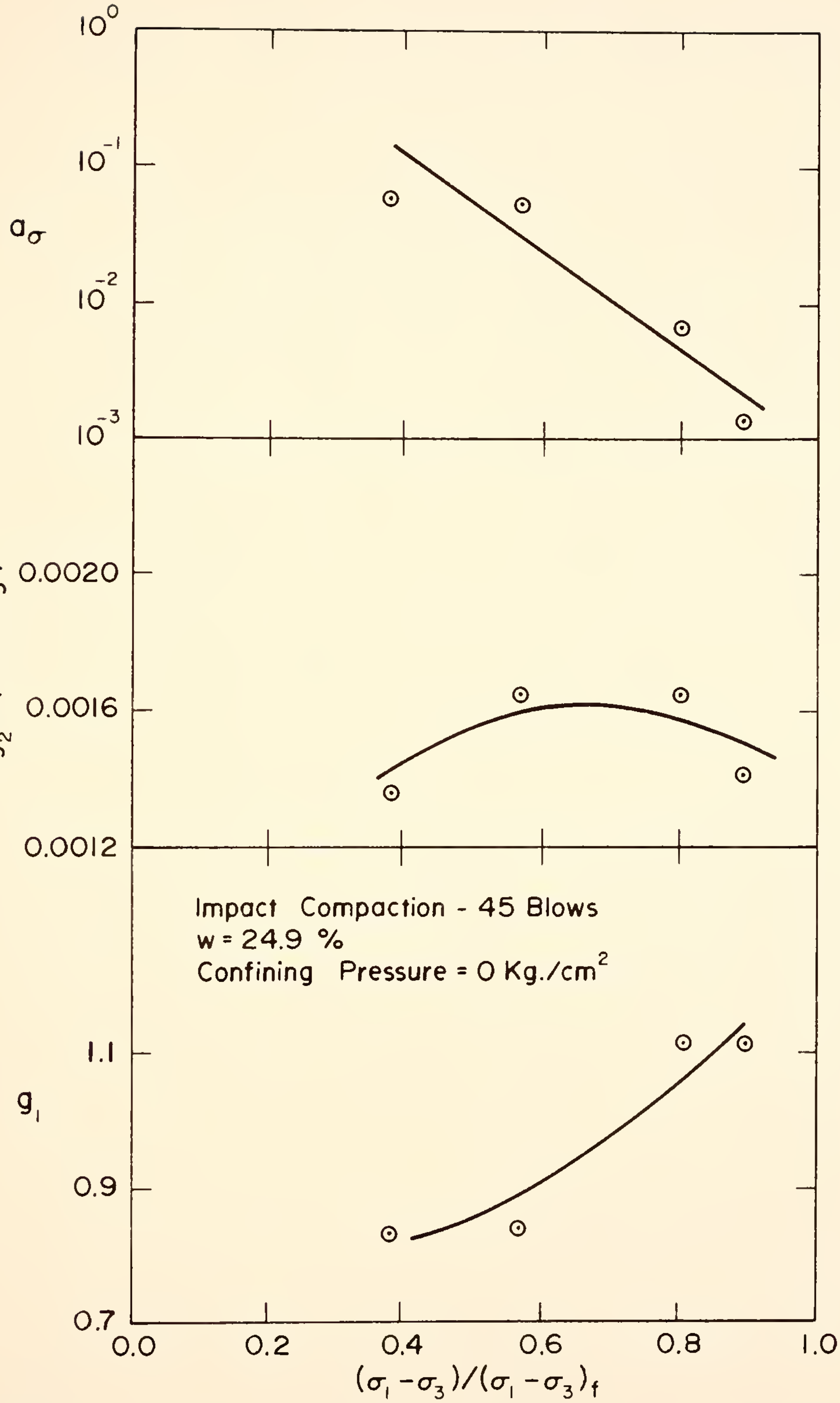


Figure 4.11.7 - Effect of Stress Ratio on Viscoelastic Parameters for EPK

the computed creep and recovery curves were obtained. It is to be noted that n value used for prediction purposes was the same for all stress levels and was the average of n values for different stress levels for the standard sample.

Figure 4.11.8 shows the predicted creep and recovery curves along with the actual experimental curves of an unconfined sample which was compacted by kneading compaction for which the values of g_1, g_2k and a_σ were obtained from Figure 4.11.6. An average n value 0.035 was used for all stress levels. The maximum error in prediction was about seven per cent at the largest stress. The recovery curves for the two stress levels, for which good curves were available, were found to agree in a satisfactory manner.

Figure 4.11.9 shows the comparison between actual and predicted creep and recovery curves for a sample which was compacted by impact and tested under a confining pressure of 1 kg/cm^2. The predicted curve was obtained using values of g_1, g_2k and a_σ from Figure 4.11.7, which data came from tests on an unconfined sample. For the only test for which full creep and recovery data were available, the agreement was satisfactory.

These results suggest the usefulness of the nonlinear viscoelastic characterization of compacted cohesive soils.

Description of these results in terms of octahedral stresses and strains is a natural extrapolation. Because of the fact that in the triaxial compression test, octahedral stresses and strains are proportional to the maximum shear stresses and strains (Equations 2.1.5 - 2.1.9), such an extrapolation must be tested by some independent method. This is in progress and will be discussed in a subsequent report.

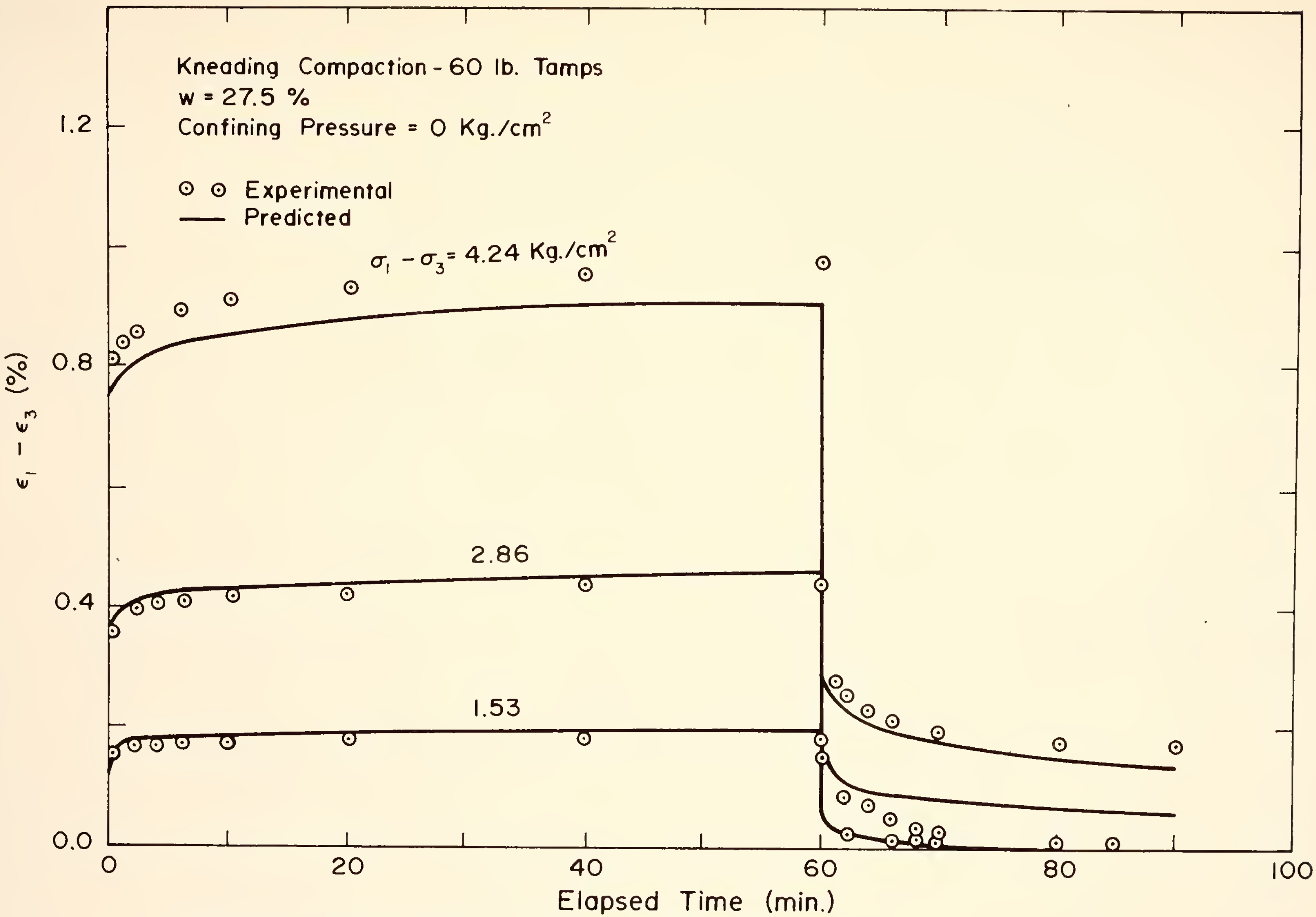


Figure 4.11.8 - Creep and Recovery for EPK

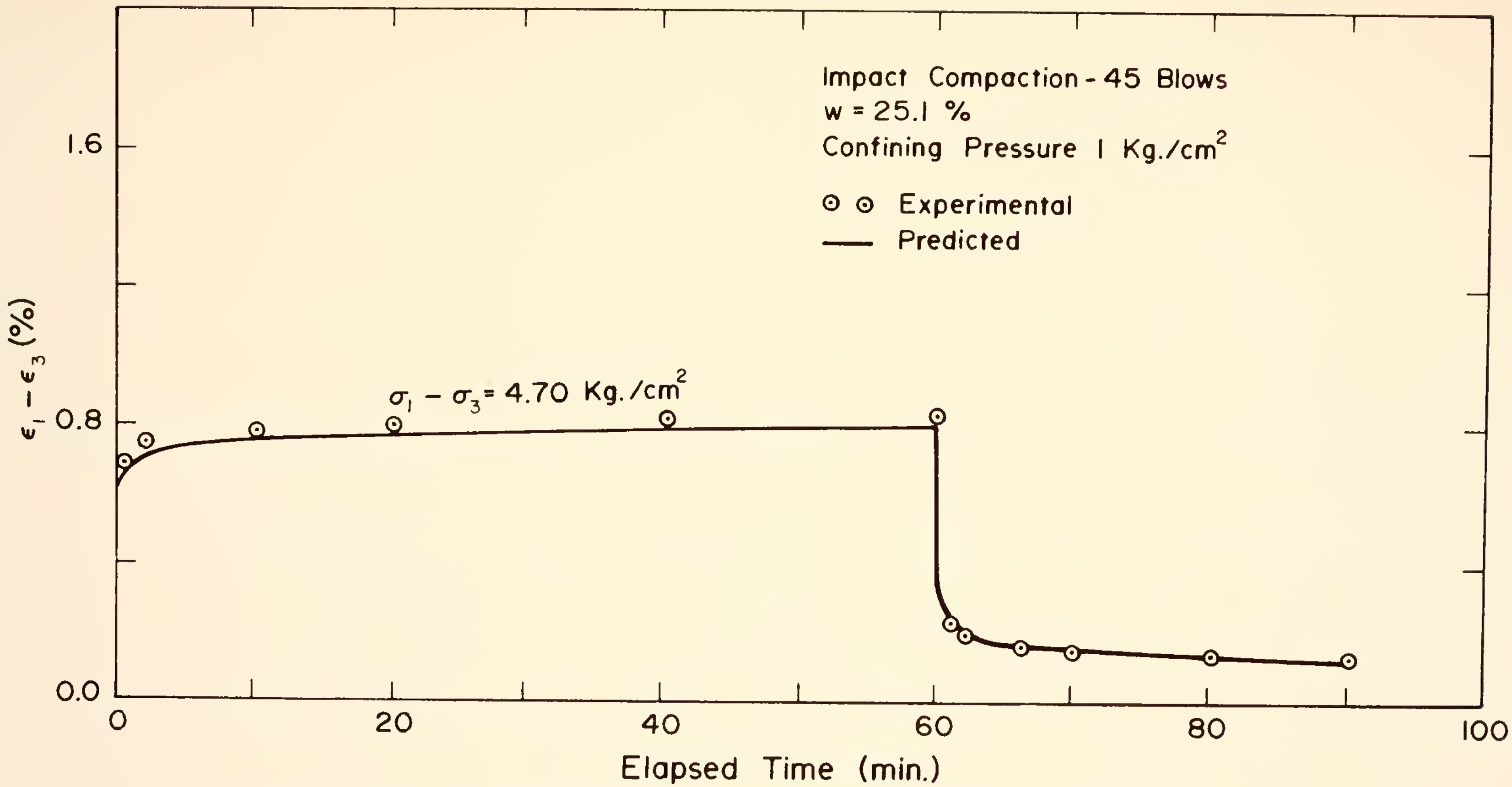


Figure 4.11.9 - Creep and Recovery for EPK

SECTION 5 - CONCLUSIONS

On the basis of the test results and analyses described herein, the following conclusions have been drawn concerning the time-dependent deformation of compacted cohesive soils:

1. The response to the second or subsequent load cycles is the most appropriate for extrapolation to prediction of deformation of a compacted embankment. All other conclusions pertain to second or subsequent load cycle response.
2. Creep behavior of compacted clays is influenced by compaction water content, confining pressure and creep shear stress level. The effects of the first two factors can be satisfactorily accounted for by expressing the creep shear stress at the ratio of the stress to the shear strength of the soil.
3. Extrapolation of the material characterization obtained from creep tests on small scale specimens of compacted clays to large scale bodies does not require consideration of a scale factor so long as air voids are continuous in the soil and moisture movement is unimportant.
4. Laboratory compacted clays exhibit anisotropic mechanical properties in creep. It is not clear whether field compacted clays would possess this attribute to the same degree.
5. The material response in a triaxial compression creep test can be characterized by a nonlinear constitutive law derivable from the principles of irreversible thermodynamics. The constitutive relation obtained is suitable for incorporation into an analysis of deformations of a large-scale embankment.

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